THE EMERGENCE OF COMPLEXITY IN A COMMON SCOTCH ROLLER

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Abstract

The fracture is a very complicated phenomenon and its dynamics is not well described, until today, by a consistent physical and/or mathematical model. In this paper we synthetically present the main experimental and theoretical results for the peeling of an adhesive tape, i.e. a viscoelastic dissipative system, viewed as a two dimensional fracture propagation. Varying the control parameter the crack dynamics appears stable in a first branch of the state curve, unstable in a second branch with stick-slip propagation, and finally again stable, but very rapid, in a third region. The unstable stick-slip dynamics becomes more and more irregular increasing the control parameter and exhibits different behaviors with transitions between periodic, aperiodic and disordered regimes. In an experiment recently performed at ESPCI-PMMH (Paris, France), the emergence of hierarchical structures in a broad range of time scales was observed in a definite region of the stick-slip regime, and this is one the indicators commonly used speaking of complex systems. At last, we underline that we do not still have a complete mathematical description of empirical data and that we lack a physical model able to explain the observed complex dynamics.

Introduction

Our physical world is no longer symbolized only by the stable and periodic orbits, with the harmonic oscillator and modes paradigm, that are at the heart of classical Newtonian mechanics. A new world of instabilities and fluctuations, which are ultimately responsible for the amazing variety and richness of the forms and structures that we see in nature, is included in the horizon of natural philosophy from about thirty years (Gallagher and Appenzeller, 1999). Surely, if we want to understand for example the nature of earthquakes, the weather variations, the growing of threes, the origin and the evolution of life, the reductionist paradigm, i.e. the explanation in terms of elementary components, is powerless. In a first qualitative sense, we can call this world “complex” and define “science of complexity” the set of experiments, models, theories, paradigms that contribute to study these phenomena. More precisely, three disciplines have modified our outlook on the physical world: statistical mechanics (in particular the non-equilibrium physics with phase transitions), the modern theory of dynamical systems, and the information theory, along with an exponential growth of the computer performance (Parisi, 1992; Livi et al., 1986). One of the more significant properties that characterize the complex physics is nonlinearity. Essentially, nonlinearity means that some effect (reaction) is not proportional to its cause (action) and therefore we cannot apply the superposition principle. This implies that the behavior of a considered system cannot be described in terms of elementary components, i.e. it cannot be studied as a “simple” system. Moreover, nonlinearity generally implies a great difficulty in solving the equations that represent the system. Even if we think that it would be possible to exactly know the initial conditions (and this is not the case, because the position for example is a real number, described by an infinite numeral succession, that means either an infinite time or an infinite quantity of information), usually nonlinear equations don’t have analytic solutions. Furthermore, the initial condition sensitivity which is proper to many systems, even with few degrees of freedom, produces very quickly a completely unpredictable time evolution of the system, despite its deterministic nature, and the orbits become chaotic. On the other hand, we have systems with a very large number of components that can be understood in a simple manner (for example a perfect gas) if we choose the right level of description (statistical and/or thermodynamical for the gas). Nonlinear processes are ubiquitous in nature and most phenomena can be studied in this “complex” optics, not only in physics, but also in chemistry, biology, neuroscience and in fields outside natural science such as economy, sociology, psychology, and cognitive science (Nicolis and Prigogine, 1989). But what does it mean, in natural philosophy, complexity? In the literature one can find many different definitions, qualitative, philosophical, quantitative (for example the Kolmogorov algorithmic complexity), but, as befits its name, the science of complexity lacks a simple and univocal definition. In a generic sense, you can refer to systems that operate at the edge of chaos and/or that are in an intermediate state between perfect order and complete disorder (Livi et al, 1988). Notwithstanding these difficulties over formal definition, we have some properties and characteristics generally assumed as indicators of complexity: power laws, degree of mixing, entropy, thermodynamic functions, cellular automaton representations, etc (Peliti and Vulpiani, 1987). But one of the most meaningful signatures of complexity is the presence of a hierarchical organization, i.e. the emergence of hierarchical structures over a range of scales. Furthermore, we will speak of self-organization (Yates, 1987) whenever the iteration of few basic rules produce the emergence of structures having features not shared by the rules themselves. It should be clear at this point that until today a "theory of complexity" does not exists. We do not have general principles or laws or equations underlying the whole complex world from which we can derive the behavior of the single specific complex system. Each complex system must be investigated in its proper way even if some features, equations and models as for example the strange attractors with fractal geometry, the logistic map with bifurcations, the cellular automata, the neural networks, can be applied to study different phenomena, physical, but also biological or human. At this step of our scientific understanding of the complex world, we could consider the "theory of complexity" a sort of theory of modeling, so defining heuristically a complex system as a system which is intrinsically hard to model, no matter which is its nature, physical or biological or another one, and no matter which mathematical or experimental tools are used (Badii and Politi, 1997). We underline that this heuristic approach does not give an answer to the fundamental question: why, if the basic physical laws are relatively simple, the world is full of so complicated phenomena? For example the fracture process, which is the object of our investigation in this work, is one of the most complicated phenomena of the physical world (Sethna et al., 2001). Fracture results of the interplay between the creation of new interfaces and the elastic deformation of the bulk material. While the creation of new interfaces is dominated by the properties of elasticity of the surrounding medium and the amount of accumulated strain energy, the constitutive properties of the medium are strongly affected by the fracture propagation. In the full three dimensional fracture of a brittle material (which is commonly referred to as "rupture"), the medium generally undergoes a progressive process of diffused damaging which then spontaneously concentrates into some region that is gradually crushed into fragments that slide and roll on each other, involving a great deal of different physical phenomena such as friction, adhesion, and plastic deformation (Atkinson, 1987; Scholz 1990). Modeling such a complicated mixture of phenomena is almost hopeless even with the spreading power of modern computers (Main, 1996). Some simpler context must be chosen where a smaller number of phenomena are considered along with a simplified geometry. The peeling of an adhesive tape provides an excellent example since the phenomenon is reduced to the propagation of a single coherent fracture front along a predetermined bidimensional interface (Aubrey and Sheriff, 1980). Moreover, the dissipative nature of the viscoelastic systems has the significant effect of stabilizing the fracture dynamics. Even with these simplifications, the phenomenon remains highly nonlinear and the dynamics shows a variety of instabilities and structures that suggest a possible underlying complexity. Furthermore, the peeling of an adhesive tape can be easily set up in experiments that provide very long data series from which it is possible to extract useful information on the nonlinear features of the system. A last remark is that the physics of complex systems is very young and some research programs that today seem fruitful might eventually in the future prove to be cul-de-sacs.

1) The fracture dynamics and the peeling of an adhesive tape

Many phenomena like materials failure, granular dynamics, earthquakes, convection in granular flow induced by vibrations, fracture dynamics and the peeling of an adhesive tape have generic nonlinear features. In particular, one of the essential characteristics common to all these systems is that they require a threshold to initiate the dynamical process and this introduces a very strong nonlinearity. In this section we will focus on the crack propagation in elastic solids and viscoelastic systems, giving some general concepts and formulae to study the fracture dynamics, the peeling and the stick-slip regime (Barquins, 1994).

The crack propagation in elastic solids is dissipative, usually irregular and accompanied by the emission of energy with characteristic noise (Maugis and Barquins, 1978). If we have two solids in contact and we want to describe the adhesion or separation between them (Figure 1), we can use the equation

$$w = \gamma_1 + \gamma_2 - \gamma_{12} \tag{1}$$

where γ_1 and γ_2 represents the free surface energies, γ_{12} the bound surface energy, and w is the Dupré adhesion energy; obviously if $w > 0$ we have adhesion, i.e. some energy is needed for creating a free surface. The second relevant observable is the released energy; more precisely, the strain energy release rate $G = \frac{\partial U_M}{\partial A}$, i.e. the amount of mechanical energy ΔU_M released by the system when the fracture surface advances of ΔA, for $\Delta A \to 0$ (we observe here that we have reduced the fracture problem to a bidimensional one).

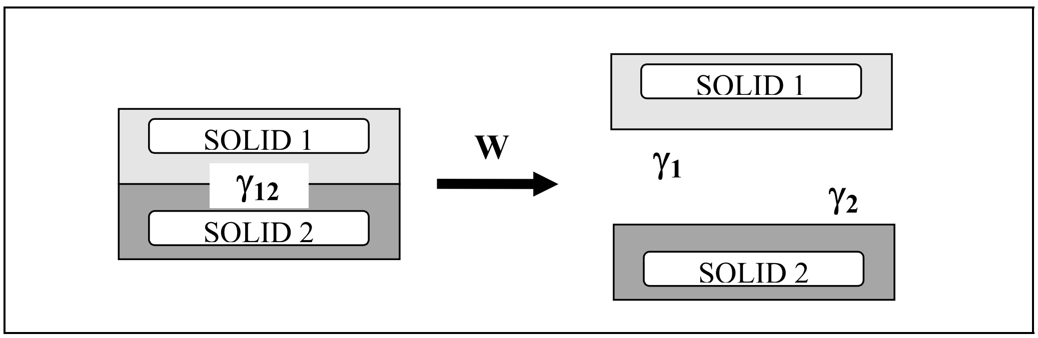

Figure 1 – Energy balance for the adhesion of two solids

For

$$G = w \tag{2}$$

the crack is in an equilibrium state that can be stable or unstable if G rises or drops following an hypothetical advancement of the fracture. When $G > w$ the crack propagates spontaneously giving rise to highly dissipative phenomena due to the elevated stress near the crack tip. As a result, a constant crack velocity is attained which is a function of the difference between G and w. Equation (2) can thus be extended to the dynamic case by adding a dissipative term such as

$$G = w + w\varphi(a_T \mathrm{v}) = \Phi(\mathrm{v}) \tag{3}$$

where v is the crack velocity, φ is a phenomenological relation that depends on the scaled velocity $a_T \mathrm{v}$, with a_T depending from temperature (following the WLF equation for viscoelasticity (Ferry, 1980)). We underline that the dissipative term is proportional to the adhesion energy and that φ depends only on the temperature T and the crack velocity v. In this context we can trace a curve $G = \Phi(\mathrm{v})$, that is represented in logarithmic scale in Figure 2.

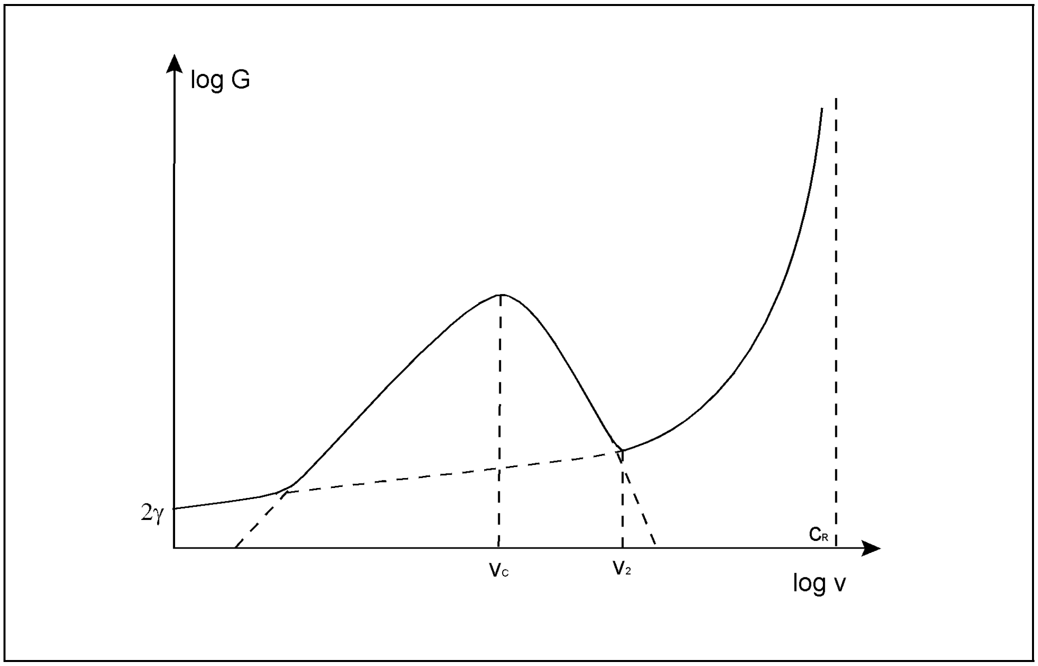

Figure 2 – Characteristic curve $G = \Phi(\mathrm{v})$

This curve can be viewed as the superposition of a term representing surface and kinetic energy with G monotonically increasing with v and diverging at the limit Rayleigh wave velocity, and a broad peak due to viscoelastic losses.

We refer to peeling when a thin adhesive film is separated from a rigid substrate. Peeling exhibits a rich and interesting dynamics with stable or unstable regimes, depending on the value of the control parameters. Two different types of peel tests are generally used to investigate the dynamics:

a) the film is peeled apart from flat rigid substrate (Figure 3);
b) the film is wound to a reel and the peeling is accompanied by rotation of the reel (Figure 4);

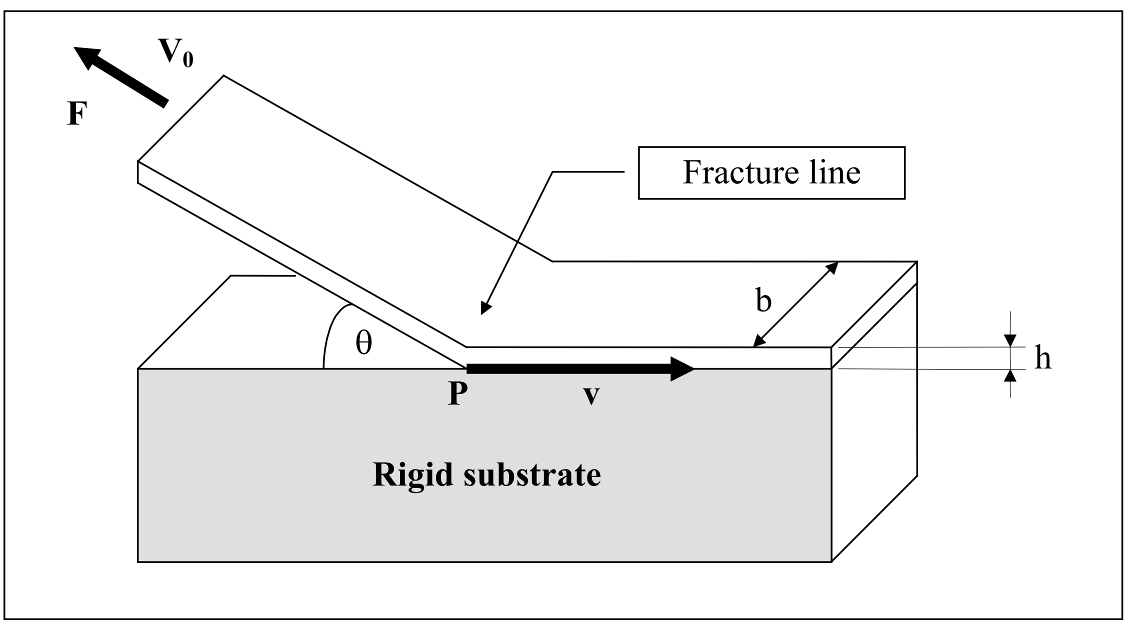

Figure 3 – A film of width b and thickness h is peeled apart from a flat rigid substrate. The force F makes an angle θ (peeling angle) with the rigid substrate.

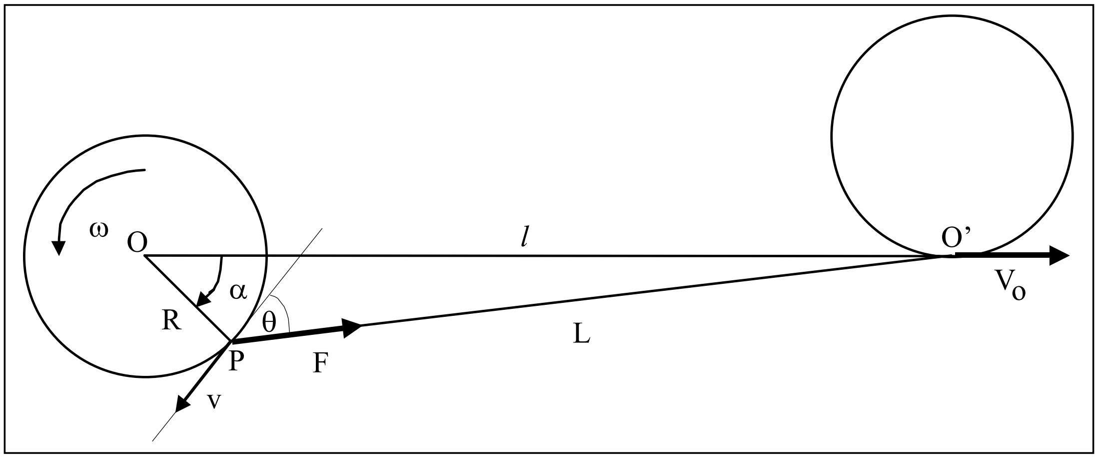

Figure 4 - The film is wound to a reel of radius R rotating with an angular velocity ω . The apparent position of the fracture is indicated by the angle α. V_0 is the traction velocity at point O' at a distance L from the fracture front. θ is the peel angle.

In this paper we will discuss above all the second experimental fracture model.

The relation between the fracture dynamics and peeling was established by Kendall (1975), which linked the pull force F applied to the free end of the film to the strain energy release rate G obtaining the equation:

$$G = \frac{F}{b}(1-\cos\vartheta) + \left(\frac{F}{b}\right)^2 \frac{1}{2Eh} \tag{4}$$

(for the meaning of symbols see Figures 3 and 4, E being the Young modulus). The first linear term is related to the geometric configuration, and the quadratic one derives from the strain energy of the new peeled film. The Kendall equation is well established since it derives from the conservation of energy and is furthermore confirmed by experiments.

If we define $F_0(\mathrm{v}) = \frac{\partial U_M}{\partial x}$ as the adherence force, we can write

$$G = \Phi(\mathrm{v}) = \frac{\partial U_M}{\partial A} = \frac{1}{b}\frac{\partial U_M}{\partial x} = \frac{F_0(\mathrm{v})}{b} \quad \Rightarrow \quad F_0(\mathrm{v}) = b\Phi(\mathrm{v}) \tag{5}$$

For the peeling angle $\theta > 30^\circ$ (which is common in stick-slip dynamics) the equation (5) reduces to

$$G = \frac{F}{b}(1-\cos\vartheta) \quad \text{or} \quad F_0(\mathrm{v}) = F(1-\cos\vartheta) \tag{6}$$

that relates univocally the adhesion force $F_0(\mathrm{v})$ to the pull force F. This is very important because we lack a direct knowledge of $F_0(\mathrm{v})$ and moreover we don't have a microscopical model for it. The equation (6) is a sort of state equation, i.e. it was derived in equilibrium conditions. However, it is expected to work also if the evolution of the variables is not too rapid in relation to some characteristic time which is still not estimated. So its applications in highly dynamical conditions and in presence of strong nonlinearity is very delicate and probably not completely correct in order to describe the instability propagation.

At last, the stick-slip (or run-arrest). We show a simple stick-slip model in Figure 5. The spring extremity A is pulled with constant velocity V with μ_d and μ_s being respectively the dynamical and static friction coefficients. At t = 0, the mass m is at rest (stick-state) and the spring is extended proportionally with V. The static force is equal to kx (k is the elastic constant, x the elongation of the spring) and increases with x up to $mg\mu_s$. At this point the slip begins obeying to the following equation:

$$m\ddot{x}+kx=mg\mu_d\,,\quad \text{with}\quad x(0)=mg\mu_s/k\ ,\quad \dot{x}(0)=V \tag{7}$$

the solution of which is

$$x(t)=\frac{mg}{k}\left[(\mu_s-\mu_d)\cos\omega t\right]+\frac{V}{\omega}\operatorname{sen}\omega t+\frac{mg\mu_d}{k}\,,\quad \text{with}\quad \omega^2=\frac{k}{m}$$

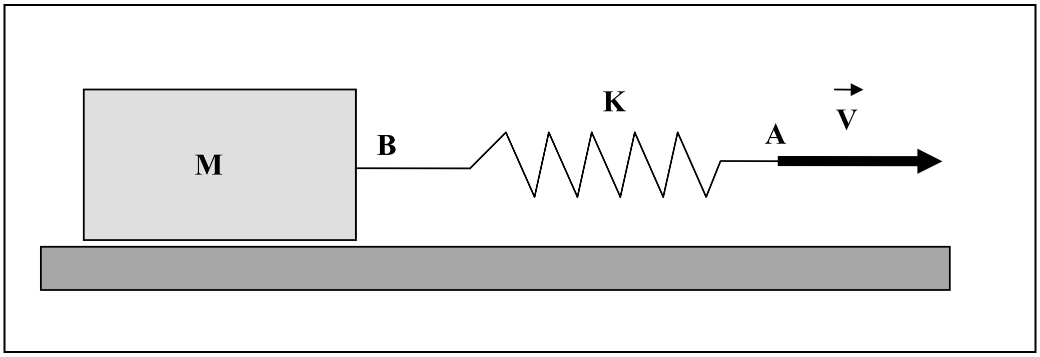

Figure 5 – Spring block slider on a flat surface

The mass arrests again for $\omega t=\pi$ (considering V very small with respect to the slip speed) with $x=mg(2\mu_d-\mu_s)/k$ and we have a new stick phase. We can also calculate the characteristic times of slip T_{SLIP} and of stick T_{STICK}:

$$T_{SLIP}=\pi\sqrt{\frac{m}{k}}\ ,\qquad T_{STICK}=2mg(\mu_s-\mu_d)/kV\,,\quad \text{where } T_{STICK}>>T_{SLIP} \tag{8}$$

The stick-slip is observed in a variety of phenomena like rock friction and earthquakes, or tearing of rubber and crack propagation in epoxy resins, etc. And also in the peeling of an adhesive tape. This intermittent motion (or self-sustained oscillations) is created by a mechanism that generates cycles of crack growth (or sliding) instability followed by subsequent arrest. The stability and instability alternation in peeling is produced by the competition between the change in the driving force (or energy release rate) and the change in the crack-growth resistance. In the next section we will describe some experiments performed in order to understand the peeling dynamics and we will give the main obtained results.

2) The previous main experiments and the empirical results

In general, the experiments on the peeling of an adhesive tape were performed utilizing two possible different set-ups. In the first the peeling was studied when a constant traction velocity V_0 is imposed onto the free end by the action of an electric motor (Figure 4). In this case, with a fixed geometry, V_0 is the only dynamical control parameter, and the limit between the adhesive tape ribbon and the free tape may be seen as a crack tip propagating with speed v. In a second type of experiment the peeling is studied when a constant applied load is clamped to its extremity (Figure 7) and the control parameter is the imposed force.

Barquins et al. (1986), Maugis and Barquins (1988), performed a series of experiments in the first above described setup. In these experiments an adhesive roller tape of radius R was unwound at a given linear velocity V_0 (up to 20 m/s) by a couplemeter motor allowing the peel force to be measured. In a modified version the winding roller was mounted on an elastic plate, the deflection of which was used to measure the peel force. The observed peeling dynamics exhibits the following behavior: at slow traction velocity the tape is peeled regularly and the dynamics is stationary; at high velocity the dynamics is also regular, but very rapid; in the intermediate range of V_0 a stick-slip phenomenon appears, the peeling of the tape being jerky with emission of a characteristic noise. Moreover, an empirical $G(\mathrm{v})$ curve was traced (Figure 6) showing that the strain-energy release rate varied as a power law of the crack velocity v:

$$\Phi(\mathrm{v}) = w + wa(T)\mathrm{v}^{n_1} \qquad n_1 = 0.35 \qquad \text{for the first stable branch}$$

$$\Phi(\mathrm{v}) = G_C\left(\frac{\mathrm{v}}{\mathrm{v}_1}\right)^{n_2} \qquad n_2 = 5.5 \qquad \text{for the second (rapid) stable branch.}$$

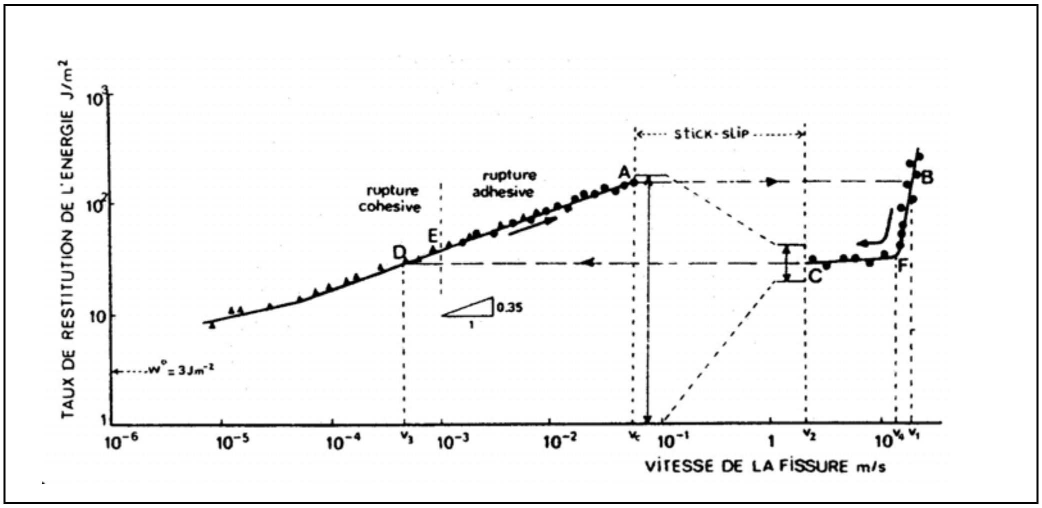

Figure 6 – Empirical $G(\mathrm{v})$ curve from Barquins et al. 1986.

In an experiment where the peeling was produced by a constant applied load (Barquins et al., 1995 and figure 7) the first stable region was found to be actually metastable, an unexpected stick-slip regime appears which was related to the inertia of the falling load, and the rapid stable branch was confirmed. The most relevant result was that the average value $\langle V \rangle$ of the measured peeling velocity remains approximately constant increasing the value of the load of one order of magnitude (Figure 8).

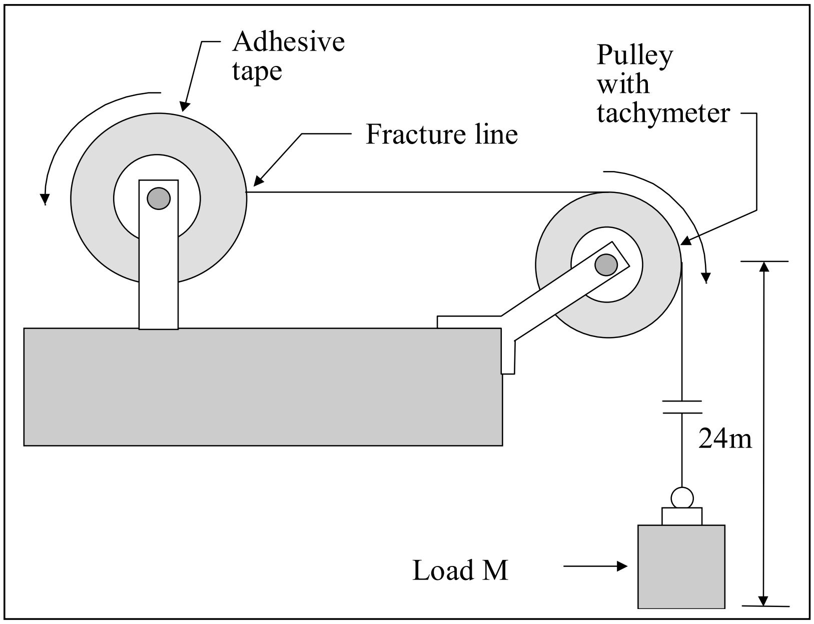

Figure 7 – Experimental set up for the peeling at constant load.

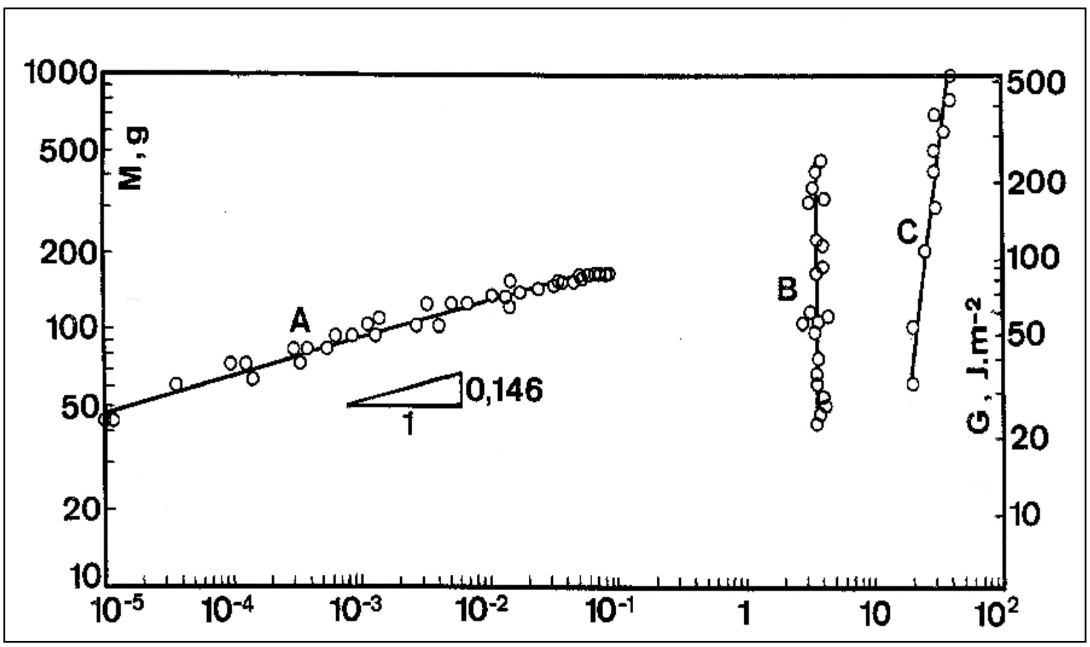

Figure 8 - Empirical $G(\mathrm{v})$ curve from Barquins et al., 1995 with vertical stick-slip branch at constant average velocity.

Obviously, in all these experiments, we must consider some influences due to temperature and humidity. Our adhesive tape is substantially composed by polymer melts, made of long flexible molecules that naturally provide the properties of sticky materials: under stress, at long time scales, they have the properties of viscous liquids, and at short time scales they deform as elastic solids. As it is well known, they depend strongly on temperature, especially near the glass transition temperature (Ferry, 1980) when the polymer transforms progressively from a viscous material to a solid. But we want to study the fracture propagation and not the phase transition of the system; if temperature and humidity don't change too drastically, they don't affect the dynamics in a significant way. More precisely, since the stick-slip dynamics is very fast (the slowest cycles have a period of 1 s), it is not affected by long term environment variations, and even the long series of events are taken in substantially unchanged conditions. However, an effort must be spent in order to provide similar conditions between different series of experiments.

3) The modelling

As a matter of fact, if one observes finely the macroscopic fracture line, he discovers that it is composed by a huge number of microfractures and microfilaments, but at present a microscopic model for the adhesion force and the crack in a viscoelastic system does not exist. This is the main difficulty in order to give a deep physical interpretation of the investigated phenomenon, i.e. to understand, describe and explain the fracture evolution. So we are constrained at the macroscopic level and generally the authors model the system by means of dynamical equations.

The first model (Barquins et al, 1986) only takes into account the elastic degree of freedom, writing the equation:

$$\dot{G} = -\frac{k}{b}\left(\mathrm{v} - \mathrm{V}_0\right) \qquad \text{with } \theta = \frac{\pi}{2}\ , \quad G = \Phi(\mathrm{v}) = \frac{F}{b} = \frac{Eh\delta}{L} = k\frac{\delta}{b} \tag{9}$$

where δ is the elongation of the free portion of the adhesive tape. The equation (9) explains the stable branches of the curve G(v), where the fixed points are v = V_0 , $G = \Phi(V_0)$, but it is not able to describe the stick-slip domain, unless speed jumps are artificially introduced. A common method to model the dynamical systems which have an unstable and/or irregular behavior is increasing the number of degrees of freedom. Following this, a second step was to add the roller inertia (Maugis, 1987; Maugis and Barquins, 1988). If we assume $G = \Phi(\mathrm{v}) - \frac{\partial U_k}{\partial A}$, $U_k = \frac{1}{2} I\omega^2$, with I momentum of inertia, we can write the equations:

$$\begin{cases} \dot{G} = -\frac{k}{b}\left(\mathrm{v} - \mathrm{V}_0\right) \\ \dot{\mathrm{v}} = \frac{\mathrm{b}}{\mathrm{m}}\left[G - \Phi(\mathrm{v})\right] \end{cases} \qquad \text{with } \theta = \frac{\pi}{2} \text{ and } m = \frac{I}{R^2} \tag{10}$$

which represents a two variable model. Choosing $\left[V_0, \Phi(V_0)\right]$ as the origin and letting $x = \mathrm{v} - \mathrm{V}_0$ we can write $F(x) = \Phi(\mathrm{v}) - \Phi(V_0)$, $f(x) = \frac{\partial F}{\partial x}$ and so the equation (10) becomes the well known Lienard equation:

$$\ddot{x} + \mu\omega f(x)\dot{x} + \omega^2 x = 0 \tag{11}$$

which typically has limit cycles in the branch with negative slope. By linearization one obtains Hopf bifurcation at points A and C (see fig 9(a)) where stable stationary equilibrium gives way to limit cycles (see Figure 9(b)). In figure 10 we can see that increasing the value of the control parameter V_0 the orbits go out of the quadrant, i.e. the solutions are not physical (Lunedei, 2001). More precisely, the two variable model produces results fitting the experimental data only when applied to the initial part of the stick-slip region where the phenomenon is periodic. But when increasing the traction velocity the self-sustained oscillations become more and more irregular, our equations (10,11) are unable to describe and to predict the observed behavior.

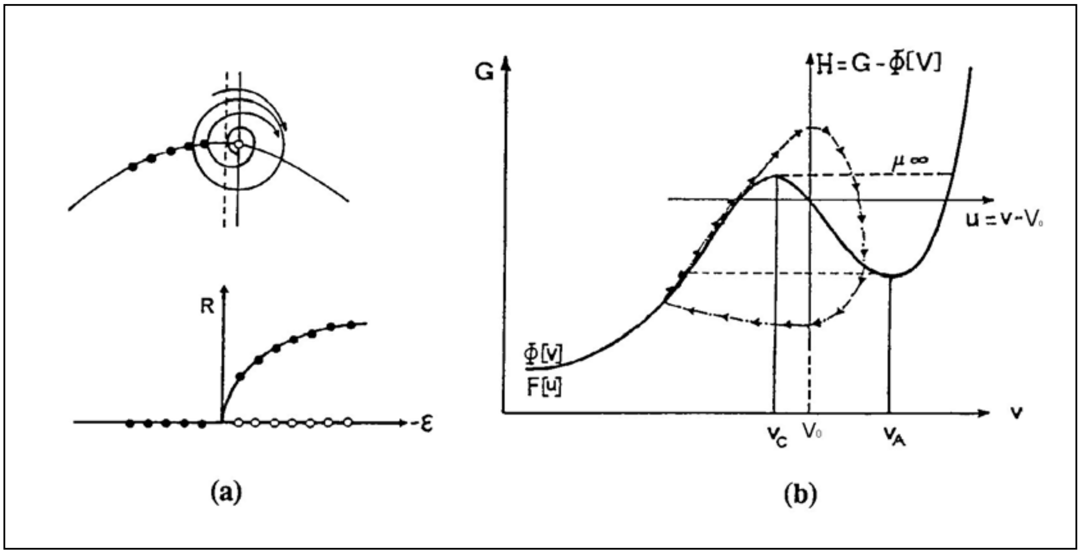

Figure 9 (a) – supercritical Hopf bifurcation (b) limit cycle over the G(v) curve.

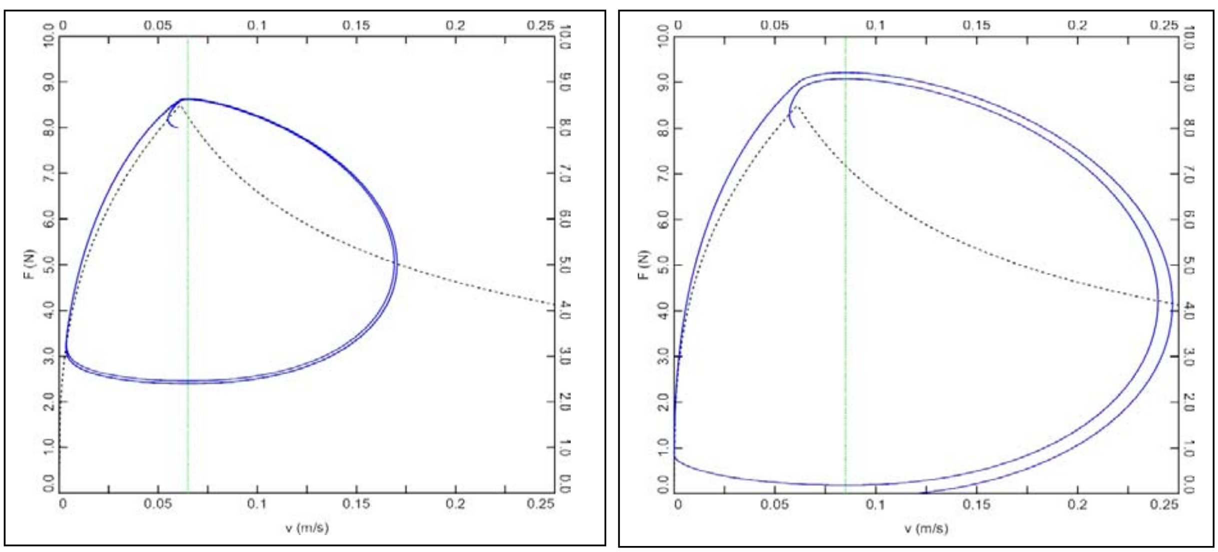

Figure 10 – Simulation of the limit cycles on the empirical G(v) curve. For incresing V_0 the cycle becomes larger and eventually it reaches the border of the positive quadrant (Lunedei, 2001).

Maugis and Barquins are quite conscious of this: "the model is more complicated when the variation of the peel angle is taken into account, which gives a third degree of freedom (…) allowing a road to chaos when limit cycles are changed into strange attractors". Following this suggestion Hong and Yue (1995) add a third variable (the peeling angle θ or the position α of the crack tip) obtaining the system:

$$\begin{cases} R\cdot\dot{\alpha} = \omega\cdot R - \mathrm{v} \\ \dot{F} = -k\cdot\left[(\mathrm{v}-V_0)+(\omega R-\mathrm{v})\cos\theta\right] \\ I\cdot\dot{\omega} = F(\mathrm{v})R\cos\theta \\ \mathrm{F}(1+\alpha) = F_0(\mathrm{v}) \end{cases} \tag{12}$$

In our analysis we used the slightly different system (Ciccotti et al., 1998):

$$\begin{cases} F\cdot(1-sin\alpha) = F_0(v) \\ I\cdot\dot{\omega} = F\cdot R\cdot sin\alpha \\ \dot{F} = k\cdot\left[R\cdot sin\alpha\cdot\dot{\alpha}-(v-V_0)\right] \\ R\cdot\dot{\alpha} = v-\omega\cdot R \end{cases} \tag{13}$$

Namely, we used a different convention for the sign of some variable, we eliminated the variable θ and we did not approximate sinα with α.

Solving the equations numerically the authors affirm that chaotic orbits are present (they found three positive Lyapunov exponents). So the stick slip would be a deterministic chaotic phenomenon and the problem seems to be closed. But firstly a well defined route to chaos does not exist and furthermore we can notice that the equation $\mathrm{F}(1+\alpha) = F_0(\mathrm{v})$ is a constraint derived in stationary conditions. Therefore it is not so natural and obvious to impose it in a highly dynamical regime. Moreover, if we study the equations (13) in a more fine way (Lunedei 2001) we discover that the proposed solutions were obtained imposing jumps of the crack velocity v. Using the constraint to eliminate α we can rewrite (Ciccotti et al., 1998) in terms of a set of three equations in three variables (F,v,ω):

$$\begin{cases} \dot{F} = -k\cdot\left[\dfrac{F-F_0(\mathrm{v})}{F}\cdot(\mathrm{v}-\omega\cdot R)-(\mathrm{v}-V_0)\right] \\ \dot{\mathrm{v}} = \dfrac{1}{\dfrac{dF_0}{d\mathrm{v}}(\mathrm{v})}\cdot\left[\dot{F}\cdot\dfrac{F_0(\mathrm{v})}{F} - F\cdot\sqrt{1-\left(1-\dfrac{F_0(\mathrm{v})}{F}\right)^2}\cdot\dfrac{\mathrm{v}-\omega\cdot R}{R}\right] \\ \dot{\omega} = \dfrac{R}{I}\cdot(F-F_0(\mathrm{v})) \end{cases} \tag{14}$$

which are valid for $\dfrac{dF_0}{d\mathrm{v}} \neq 0$, $F \neq 0$ with two singular points at $\mathrm{v} = \mathrm{v}_C$, $\mathrm{v} = \mathrm{v}_A$. The numerical solutions of (14) admit a cycle only if they are forced by hand to avoid the singularities. More precisely, the solutions must be obliged to jump from one branch to the other when the critical velocities are encountered. Without that the system has no physical solution. So we argue that the deterministic chaos in the stick-slip seems to be rather artificial and not completely proved as intrinsic to the phenomenon. At this point, we can guess that the philosophy based on increasing the dynamical variables number in order to have a model able to describe the irregular stick-slip regime, is not the more suitable one. Moreover, the phenomenon of instability propagation in a viscoelastic medium could be more complicated than a "simple" dynamical system and could have some characteristics proper to a complex system (De Gennes, 1979; Kinloch and Young, 1983). This criticism has stimulated further theoretical and experimental research. The aim was to have a more accurate knowledge of the stick-slip propagation, firstly empirical. The highly nonlinear phenomena as the fracture dynamics produce really unpredictable evolutions. In order to extract useful information on the motion from experiments, we must record sufficiently long time series, i.e. sequences of data representing the time evolution of one or more observable. After that, we can use the statistical analysis, or the geometrical reconstruction of the attactors in a suitable phase space, or some other technique to investigate the dynamics. The general philosophy of this approach is to draw out a physical meaning from an empirical signal, bypassing the knowledge of the underlying dynamics and/or the corresponding equations (Eckmann and Ruelle, 1985; Ruelle, 1987). In this optics, we can search the significant points (bifurcations and so on) and eventually we can detect the emergence of hierarchical structures, one of the most significant complexity indicators (D'Alessandro and Politi, 1990). The problem will be to choose the good observable, i.e. the more proper observable to be measured with a sufficient precision and over a convenient long time. For this, Barquins, Ciccotti, Giorgini and Vallet have set up a new experiment and the first provisional results show a stick-slip behavior more complex than we could expect basing strictly on the theory of dynamical systems (Vallet et al., 2001).

4) The new experiment

The new experiment that has been set up at the PMMH – ESPCI (Paris, France) aims at a complete description of the phase space of the stick-slip dynamics and its evolution as a function of the control parameter V_0.

The experimental assembly is the classic one with constant traction velocity. This condition is enforced with the aid of a very stiff motor that enrolls the tape on a new ribbon (Fig. 4). Stable peeling is observed for traction velocities lower that a first critical velocity v_C and larger than a second velocity v_A. In the intermediate range the peeling is jerky and with rising velocity the stick-slip dynamics becomes more and more complicated.

Measuring the phase variables

We proceed now to a synthetic description of the techniques developed in order to measure and record the evolution of the fundamental dynamical observables of the experiment, namely the traction force, the rotation velocity of the reel, the apparent position of the fracture front on the reel (which determines the peeling angle), and the acoustic emissions. A picture of the assembly is shown in figure 11.

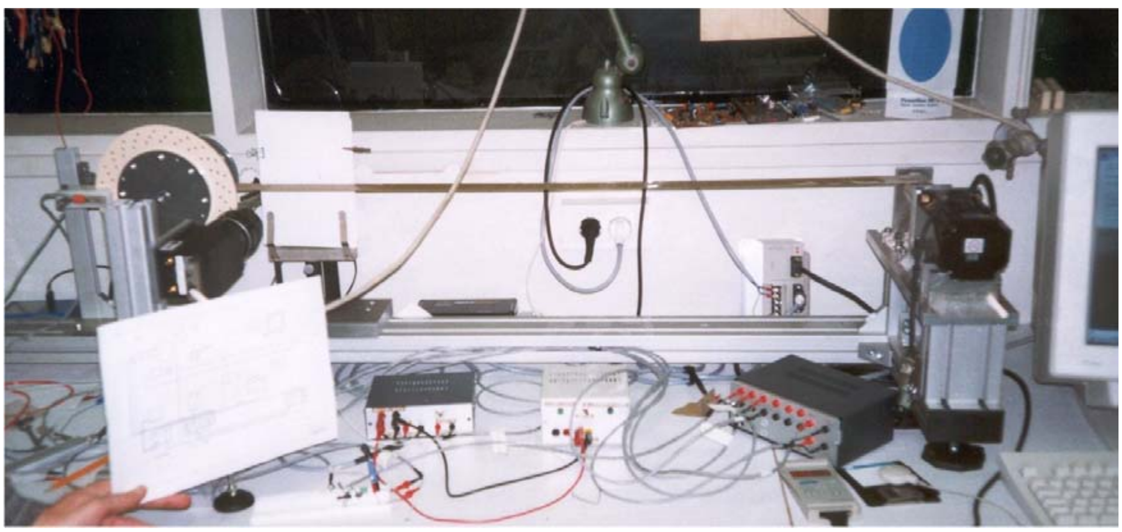

Fig 11 – General overview of the experimental set up

The rotation velocity of the reel is measured by a tachymetric wheel (Fig 12). Three shells of holes induce an ordered sequence of commutation of the three photocells. The signals are electronically combined into one three-level analogic signal that is recorded by a computer, then converted into the ordered sequence of activated photocells that allows one to calculate the rotation velocity along with its sign. Since 24 timings are available for one tour, the average velocity can be monitored with valuable resolution in time. When elevated values of the rotation velocity have to be measured, the wheel is changed with another one which has an inferior number of larger holes.

The reel of the adhesive tape is installed over a vertical steel beam that is sufficiently stiff in order not to affect the peeling dynamics, but whose small deflections in the stick-slip regime are large enough to be measured by laser ranging. The deflection is calibrated to measure the force F applied to the adhesive tape.

To evaluate the apparent position of the fracture front (denoted by the angle α), we set up a high frequency digital camera (Fig. 13) that acquires a vertical line every millisecond. The sequence is analyzed on a computer to extract the vertical position of the adhesive tape at the given cross section. This measure is then related to the apparent position of the fracture front by simple geometric relations.

At last, a high precision microphone is placed near the position of the crack front to record the characteristic noise produced by the stick-slip dynamics. The identification of the acoustic bursts associated with the slip events are the most precise method for determining the long series of inter-event times that are the main observable of the present study. The significance of the acoustic method for the recognition of events was previously tested (Barquins et al., 1995) by a correlation with the light emissions (sparks) associated with the strong ionization produced by the rapid slip and measured by a photomultiplier.

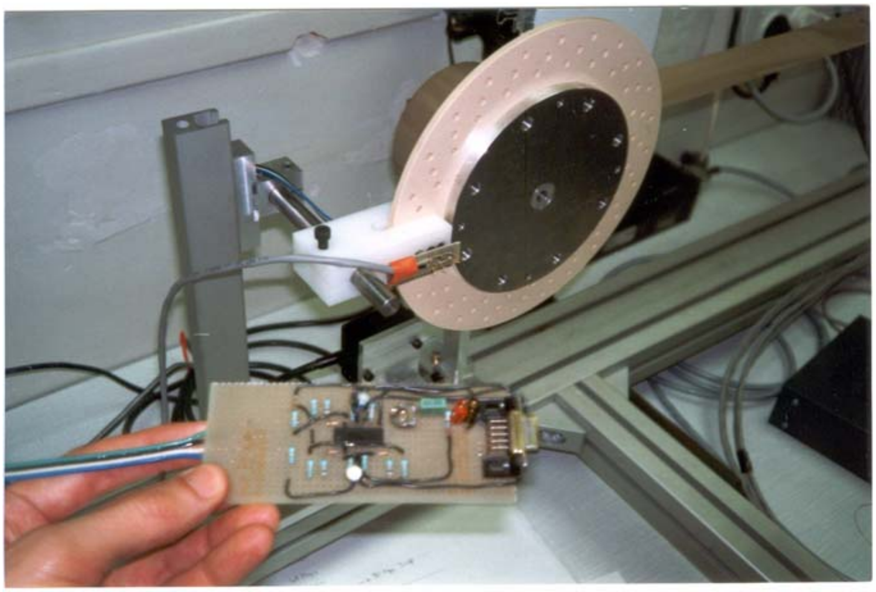

Fig 12 – A tachymetric wheel measures the evolution of the rotation velocity ω of the support. The electronic device produces a code related to the active photocells that are situated inside the white support.

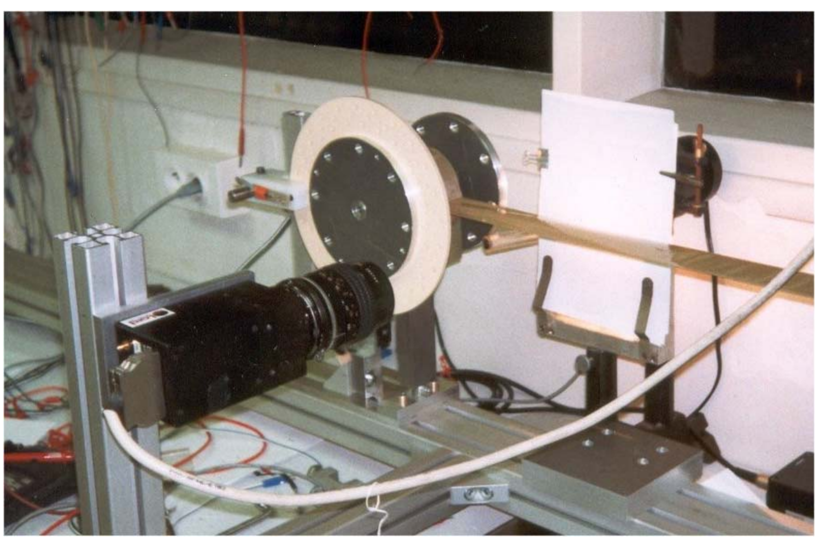

Figure 13 – The high frequency digital camera measures the vertical position of the tight tape, providing the evolution in time of the apparent position of the fracture front.

5) The results

As a first experimental step, we choose to measure the time interval between two subsequent events, i.e. our observable is the period Δt. Usually, in physics, and especially in dynamics, the time is not a variable, but rather a parameter. However, in our situation the dynamical equations proposed to describe and explain the stick-slip at least partially failed. The time interval Δt is the indicator that we can measure more precisely in order to evaluate the irregularity of the dynamics. Exactly as we do studying the behavior of a pendulum, for which we can define periodic, quasi-periodic, aperiodic regimes, only measuring the periods, even if we do not know the equations governing the phenomenon. Obviously, the knowledge of the time interval series is not sufficient to model completely the dynamics. For example, we can not discover if we are in presence of space-time chaos only by numerical time series of periods, even if these series were chaotic. But at least our philosophy can put in evidence the time structures underlying the phenomenon and give a criterion to discriminate different dynamical regimes, as a function of the control parameter.

Analysis of data

The acoustic emission of the adhesive tape in the stick-slip regime is recorded by a high quality microphone and digitized at the standard audio sampling frequency of 44100 Hz with a 16 bit signal that is stored in raw binary .wav files.

Long records of events have been acquired for many values of the control parameter, i.e. the traction velocity V_0, spanning the unstable stick-slip regime. Since the sequences may involve thousands of events, some automatic recognition process is necessary for the analysis.

For low values of the traction velocity, the stick-slip events are clearly separated and regular. They appear as abrupt acoustic bursts, followed by a gradual oscillating decay. A very simple threshold-window method is appropriate in this case: an event is detected any time the signal crosses a given threshold (evaluated as three times the root mean square of the signal), then a time window is skipped in order to let the signal decay below the threshold. Since the events are similar in intensity and well separated in time, the time window is easily chosen below the least observed interval and above the estimated decay time.

When the traction velocity grows to higher values, the events become more and more frequent and irregular both in timing and intensity, making their separation problematic. The above simple algorithm is no longer sufficient and some more elaborate criterion was developed. Since the oscillations become more persistent, the focus is shifted to the identifications of rapid changes in the signal. Due to the band filtering of the microphone, the burst is characterized by an irregular oscillation with a characteristic frequency given by the upper limit of the band (e.g. 6 kHz for the first microphone that was used). The acoustic power spectrum of the signal will be observed to rapidly fall above such a frequency. On the other hand, the abrupt initiation of the burst, is characterized by a local enhancement of the high frequency content. For that reason we performed a moving window Fast Fourier Transform on the signal and built a new index based on the integration of the high frequency acoustic power. Such an index is plotted in red in figure 14 and is evidently well correlated to the events.

A special attention has been taken in order to evaluate the misfits of the recognition method. These may be of two kinds: (1) a “missed event” results in the substitution of two proper time intervals with a false longer time interval, (2) a “false event” results in the loss of one proper time interval in favor of two false small ones. In order to ascertain that the observed structures are not due to the effects of these misfits, several set of data have been analyzed manually and presented a good agreement with the results of the above algorithm.

This method allowed us to identify the events up to a traction velocity V_0 = 10 cm/s with an efficiency better than 90%. After that, the oscillations in the signal appear to be almost continuous. A possible interpretation is that chaos and/or turbulence is completely installed (Becker, 2000). But at present we can not exclude that we have reached the limit of resolution of our apparatus, and that we are not able of distinguishing the events.

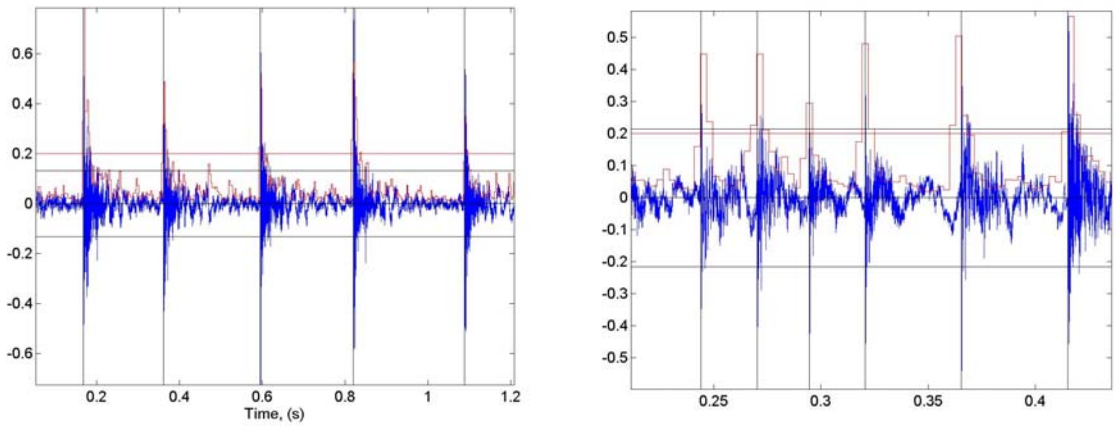

Figure 14 – Automatic recognition of the slip events in the acoustic signal. Black horizontal lines indicate standard thresholds, the red signal is the FFT based index along with its threshold.

The emergence of hierarchical structures

Although the stick-slip cycles are generally not periodic, it is interesting to plot the average period as a function of the traction velocity (see Figure 15). In first approximation the average frequency is proportional to the traction velocity and a linear fit provides the relation:

$$\nu_m = \frac{1}{\langle T \rangle} = 0.53Hz\left(\frac{V_0}{2.1mm/s}\right)$$

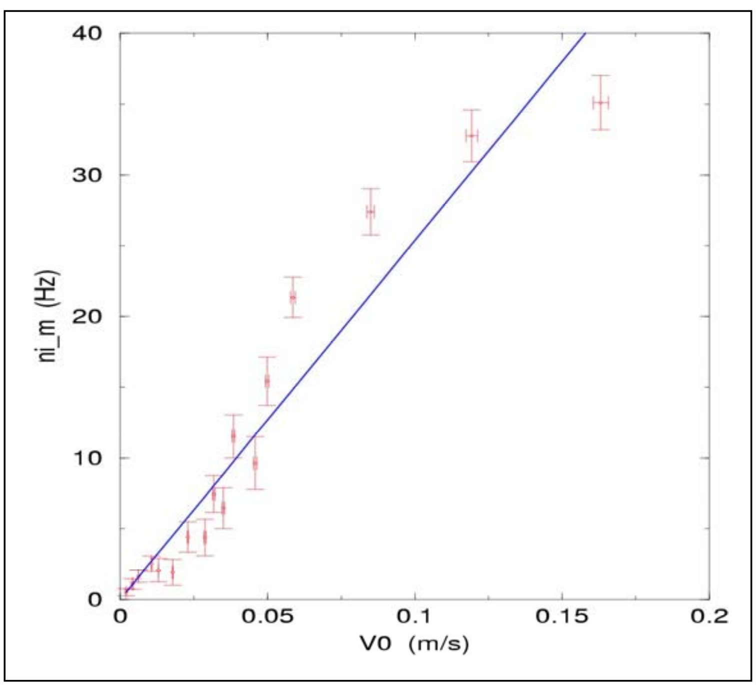

Figure 15 – Average period of stick-slip cycles as a function of the traction velocity.

Actually, the cycles are only quasi periodic for low traction velocities, while they become irregular increasing speed. The analysis of the time intervals has put into evidence that the dynamics goes through a series of progressive complications when the traction velocity V_0 passes some subsequent critical values. In particular, there is a first low velocity domain in which the cycles are approximately periodic and for which the period diminishes as the traction velocity is increased (Figure 16). This domain is followed by the appearance of sparse rapid events which have a period (here simply meaning the time interval between two subsequent events) that is a multiple of the fixed time interval 0.02s (Figure 17). The events of double or triple period become progressively more frequent leading to the establishment of an ordered structure with three possible periods, then they decrease again in frequency until a new regular regime is observed with substantially periodic events at the short time interval of 0.02s. This time interval does not change with traction velocity, until at higher speed, the cycles undergo a second bifurcation with the appearance of sparse events with a duration multiple of a new shorter fixed time interval which is about ten times shorter than the previous one. These measures are still in analysis, due to the increased difficulties of separating such a frequent series, but we can anticipate that a new structure is again developed. At higher velocities the acoustic signal is mixed to such an extent that we can not distinguish any event at the present state. However, at a traction velocity of 3 m/s the peeling becomes again stable without the crackling noise and it remains stable until the reach of the limit velocity given by the Rayleigh wave velocity in the crack surface.

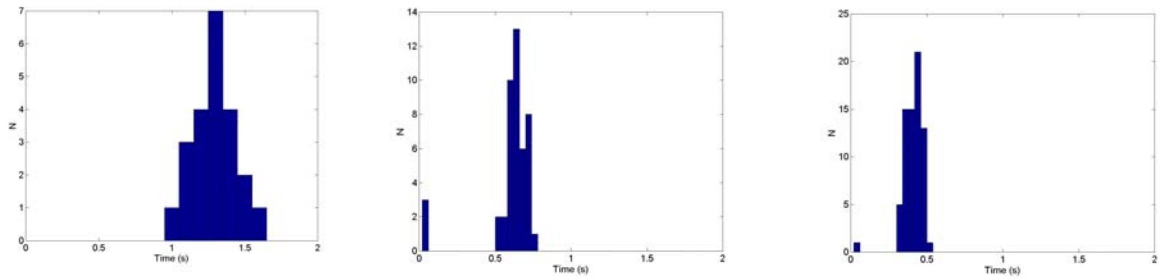

Figure 16 – Distribution of the time intervals for V_0 = 2, 4, 6 mm/s.

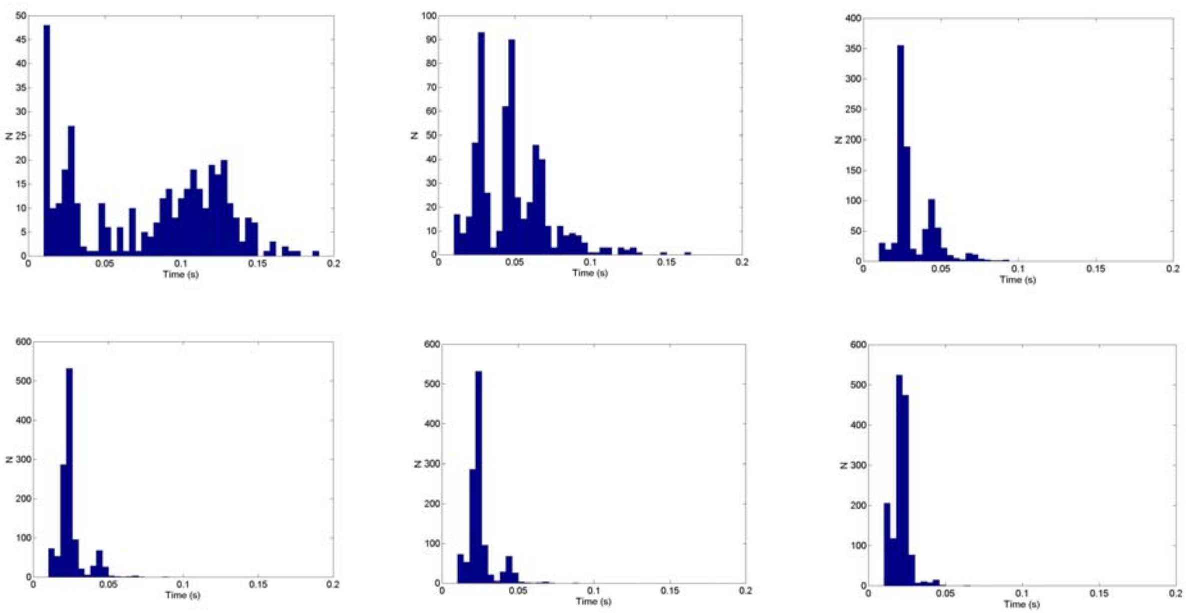

Figure 17 – Distribution of the time intervals for V_0 = 2, 3, 4, 5, 6, 7 cm/s.

We can resume complication cascade like follows:

- Up to 0.7 mm/s: stable peeling
- At 0.7 mm/s beginning of regular stick-slip. Time periods fall with growing traction velocity
- At 1 cm/s apparition of first multiplets with $\Delta t \approx 0.02, 0.04, 0.06$ s. The structure develops up to 3 cm/s, then it concentrates on the shortest interval
- At 6 cm/s apparition of sub-multiplets with $\Delta t \approx 0.001, 0.002, 0.003$ s. The structure develops, but the signal is lost at 10 cm/s
- At 3 m/s the peeling becomes again stable and humming up to the maximum traction velocity of the engine 6 m/s

Developments

Some numerical simulations of the stick-slip dynamics have been set up taking into special care the large variations of the peeling velocity during the cycles. Preliminary tests show that the simulations can accurately describe the low velocity regime and they also predict a series of bifurcations which correspond to a progressively complicating dynamics. Although the simulated dynamics appears qualitatively different than the observed one, we are performing a new set of simulations which include the effective mechanical parameters of the experiment in order to verify if at least the location of the critical values of the bifurcations are reproduced.

The data are also being analyzed with a different approach, that is by a statistical analysis of the series of time intervals (figure 18), with the aim of investigating the correlation between subsequent cycles and extracting the predictive information hidden in the data (Packard et al., 1980).

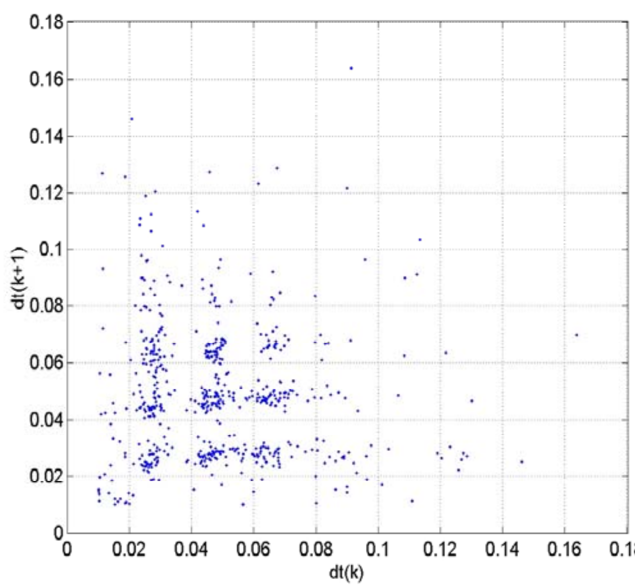

Figure 18 – Correlation between adjacent events of the three level structure (V_0 = 3 cm/s) in a phase space (t_n , t_{n+1}).

In particular, we analyzed the series of time intervals of the data with V_0 = 3 cm/s, for which the intervals only have the three multiple lengths, denoted with letters A, B, C. A simple statistical analysis shows that the subsequent intervals are not independent, but they are neither well described by a first order Markov process. We are presently evaluating the predictive power of the series and the presence of nonlinear correlations by an evaluation of the embedding dimension.

Conclusions

As we have seen, the main dynamical models constructed to explain the peeling evolution of an adhesive tape can predict correctly only the stationary behaviors and the approximately periodic stick-slip cycles. But when the stick-slip becomes very irregular the proposed models are insufficient also if we increase the number of degrees of freedom.

On the other hand, the results of new experiments show that the stick-slip dynamics is more rich and complicated than a simple bifurcation's route to chaos.

We observe hierarchical structures in a definite traction velocity range that can suggest the emergence of complexity, at least in qualitative sense, when a fracture is produced and evolves in a viscoelastic system.

Strong efforts are being spent in the direction of resolving the series of more rapid events associated to the higher traction velocities, to understand the complex dynamics which is progressively installing. The dynamical models predict a cascade of bifurcations leading to deterministic chaos. We do not have the classic cascade with Feigenbaum universality (Cvitanovic 1989), but it could be possible that the observed complex dynamics leads to time chaos. Another point of view could be statistical: for high traction velocities the peeling in the stick-slip regime could not any longer be described by dynamical equations, but would rather be the result of a large number of degrees of freedom cooperating to constitute a self-organized system in a critical state far from equilibrium. But, before venturing an interpretation, we must measure the other observables F, ω and, above all, the crack speed v. Furthermore, we must investigate the possible mathematical correlations of our time series of data. Finally, the goal would be to find a physical interpretation of the phenomenon or, more precisely, discovering the physical laws and equations that originate the observed fracture complex dynamics.

However, the peeling apparatus set up in the PMMH-ESPCI laboratory provides a good experimental model to study nonlinear phenomena in a fine way, following step by step the increasing instability and furnishing the suitable long time series of data.

At last, we want underline how a common scotch roller can be so enigmatic and full of mystery to justify the ancient saying “I know that I don't know”. Perhaps this sentence could be the deep meaning of “complexity”.

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