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RESEARCH NOTE

Differences between static and dynamic elastic moduli of a typical seismogenic rock

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SUMMARY

Static and dynamic elastic moduli of Calcare Massiccio mudstone-limestone, the typical seismogenic rock in the Italian Apennines, are measured using a standard uniaxial static compression test, a dual cantilever forced oscillation test and ultrasonic measurement of elastic wave velocities. These measurements cover nine decades in frequency including the seismic domain. Neither a significant frequency dependence nor a pronounced strain amplitude dependence was observed, providing a Young's modulus of (75 ± 7) GPa and a Poisson's ratio of (0.28 ± 0.02). These values are characteristic of Calcare Massiccio in undamaged condition.

Key words: elastodynamics, laboratory measurement, moduli.

1 INTRODUCTION

Elastic constants are relevant parameters whenever stresses and strains are considered. The two main methods used to measure them are: (a) the static method, which is based on the measurement of the deformation induced in a material by the application of a known force, and (b) the dynamic method, which implies measuring the ultrasonic body wave velocities.

As a result of inelastic effects, it is not granted that these values coincide. On a large scale, for Earth's crustal rocks only dynamic parameters can be easily constrained from the investigation of seismic waves (generally in the range from 1 Hz to 10 Hz). As a consequence, their use in modelling the tectonic deformations is ubiquitous, yet the appropriateness of this is in doubt (Jaeger & Cook 1969).

Several laboratory measurements can be found in the literature (*cf.* Eissa & Kazi 1988, and references therein), where the elastic moduli are divided into static and dynamic moduli. The proposed empirical relations show that values of static moduli appear to be in general 5 to 10 per cent lower than those of dynamic moduli. The scatter among the data is very large and in some cases values obtained from static measurements can be smaller than those from dynamic estimates by as much as an order of magnitude.

The scope of the present paper is to investigate the frequency dependence of the elastic moduli in a typical seismogenic rock in undamaged condition by using three different techniques that cover nine decades in frequency including the seismic domain. In order to provide an estimate of the different accuracy of the three techniques as compared to the scatter between samples, we performed the same measurements on a reference material, poly-methyl-methacrylate (PMMA), for which there exists a wealth of measured data over a wide frequency range.

2 SAMPLE MATERIAL

The rock analysed is the Calcare Massiccio (a compact homogeneous mudstone-limestone), which is very interesting *per se* because it is the typical rock of the seismic focal regions in the Italian Apennines (Almagro 2002), where the vast majority of Italian seismicity occurs and for which data are totally lacking. The samples we analysed come from the Casavecchia Fratelli quarry located in Cagli (Pesaro), central Italy. The chemical composition is reported in Table 1. The apparent density is $\rho = (2.71 \pm 0.03)$ kg m^{-3} and the estimated porosity is $\phi = (5 \pm 2)$ per cent. Isotropy was verified within experimental errors (1 per cent) by measuring ultrasonic wave velocity on a cubic sample of 10 cm side.

3 EXPERIMENTAL METHODS AND RESULTS

The high frequency moduli were obtained by measuring the velocities v_P and v_S of longitudinal and transverse elastic waves at 1 MHz and 75 kHz by using piezoelectric transducers (produced by Valpey Fisher, Hopkinton, MA, USA and Panametrics, Waltham, MA, USA) both in transmission and in pulse-echo configuration. These provided very consistent results with $E = (81 \pm 5)$ GPa and $\nu = (0.28 \pm 0.02)$. A detailed account of the measurements and techniques can be found in Ciccotti *et al.* (2004).

The measurements in an intermediate frequency range (0.01 ÷ 20 Hz) were performed by imposing forced oscillations on specimens of dimensions 60 × 13 × 5 mm with a dual cantilever DMA2980 machine (produced by TA Instruments, New Castle, DE, USA) and measuring the amplitude ratio and phase shift between force and displacement.* The average value of the measured Young's

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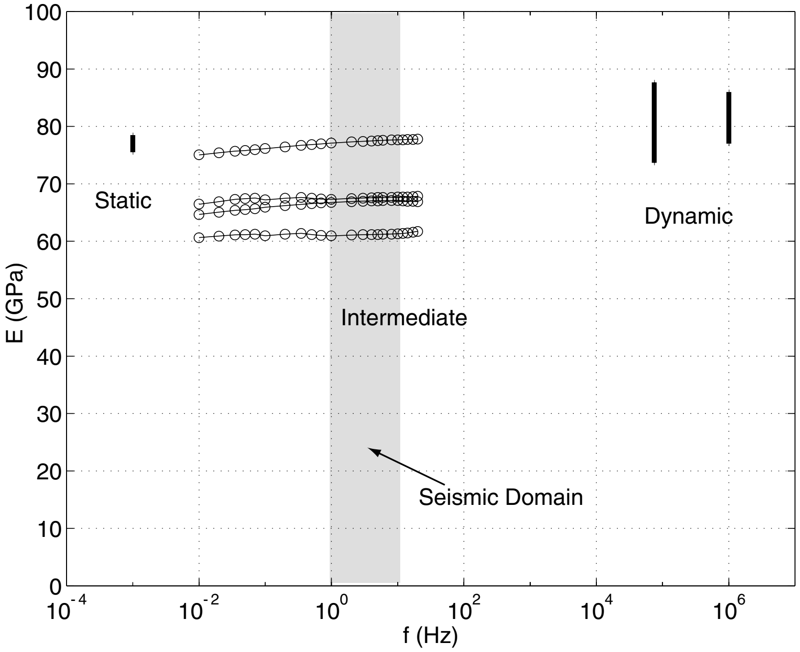

Figure 1. Static, intermediate and dynamic measurements of the Young's modulus of Calcare Massiccio. Data of four samples of the same material are reported for intermediate measurements. Range of variation is reported for static and dynamic measurements.

Table 1. Chemical composition of Calcare Massiccio obtained through XRF analysis at the Dipartimento di Scienza della Terra of Bologna, Italy. LOI = loss on ignition.

CaO	50,55 per cent
LOI	43,69 per cent
MgO	5,00 per cent
Fe_2O_3	0,30 per cent
Al_2O_3	0,20 per cent
Sr	170 ppm

modulus is $E = (68 \pm 6)$ GPa, with a very weak positive dependence on frequency (see Fig. 1). The Poisson ratio was required as an external parameter and it was assumed to have the measured dynamic value $\nu = 0.28$.

The measurement of the static elastic moduli was performed following the standard procedure UNI9724[†] by using an AMSLER (Zwick GMBH & Co., Ulm, Germany) uniaxial compression machine on prismatic specimens of dimensions 20 × 5 × 5 cm along with longitudinal and transverse strain gauges.[‡] The measured Young's modulus is $E = (77 \pm 1.5)$ GPa. The Poisson ratio is not well constrained because the measurements change between 0.30 and 0.43 on different specimens. We note that because the tests have a typical duration of about 20 min, the static measurements should properly be associated with a frequency of $\simeq 10^{-3}$ Hz.

4 DISCUSSION

At a first glance we observe that the static value of the Young's modulus is lower than the dynamic one, however, it is higher than the one in the intermediate range (Fig. 1). A possible cause could be the different strain amplitudes involved in the three tests: $\epsilon \simeq 10^{-3}$ for static measurements, $\epsilon \simeq 10^{-5}$ for dual cantilever, $\epsilon \simeq 10^{-7}$ for ultrasonic waves. However, no systematic strain amplitude effect is noticed. In fact, this is not a real problem because the intermediate values are based on a method that is known to provide very good relative precision at different frequencies on the same specimen, with a comparatively quite modest absolute accuracy as a result of problems of repeatability in the clamping of the specimens and high sensitivity to small geometrical imperfections of the sample thickness (Menard 1991). This is clearly apparent in Fig. 1 where each of the four data series, relative to different samples, shows a very weak frequency dependence within 3 per cent (the specimen is clamped only once), while the scatter between measurements on different specimens is approximately 10 per cent (repeated clamping). On the other hand, the weak trend observed in each specimen is consistent with the small difference between the static and dynamic moduli (approximately 5 per cent).

In order to investigate the role of the different accuracies of the three measuring techniques, we repeated the same measurements on a reference material, poly-methyl-methacrylate (PMMA; see Fig. 2), for which a wealth of measurements exists and which, being a viscoelastic material, exhibits a large variation between static and dynamic moduli. The static measurements gave a value of $E = (3.6 \pm 0.1)$ GPa, again higher than the lower limit of the intermediate measurements, which show nevertheless a pronounced dependence on frequency ranging from (2.7 ± 0.2) GPa at 0.01 Hz to (3.8 ± 0.2) GPa at 20 Hz. The scatter between specimens is also large (approximately 15 per cent), but smaller than the dependence on frequency for each specimen (approximately 30 per cent in this frequency range). The dynamic measurements gave a constant value of $E = (6.02 \pm 0.2)$ GPa at both 75 kHz and 1 MHz and a Poisson's ratio $\nu = (0.33 \pm 0.03)$.

While the dynamic values are very reproducible and consistent with independent measurements, which range between 6.0 and 6.3 GPa (Boudet & Ciliberto 2000; Filippi *et al.* 1999; Ferry 1980),

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[†] Ente Nazionale Italiano di Unificazione—http://www.uni.com—Standard norm: UNI 9724-8:1992—ICS code: 91.100.15. Title: Stones. Determination of the elastic modulus (monoaxial).

[‡] In collaboration with the Laboratorio di Resistenza dei Materiali, DISTART, Università di Bologna, Italy.

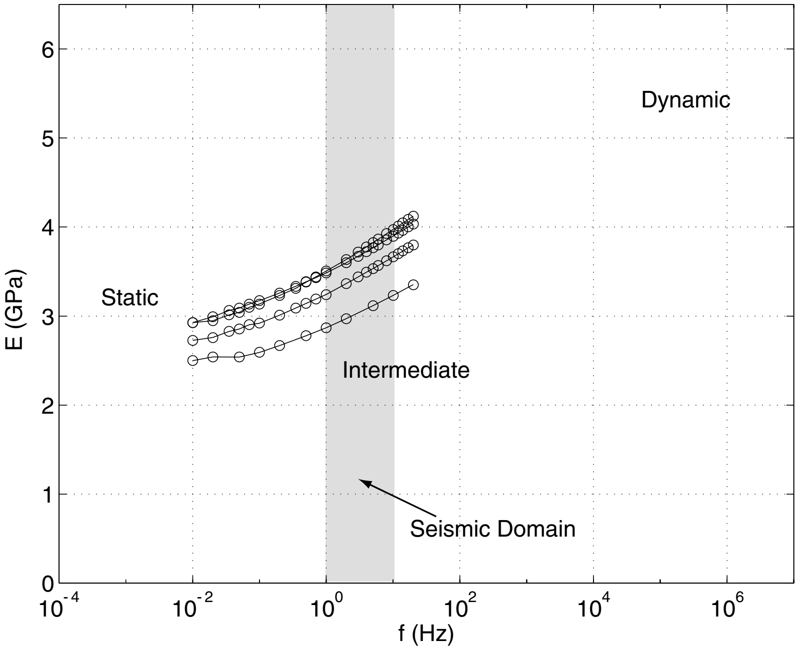

Figure 2. Same as Fig. 1 for poly-methyl-methacrylate (PMMA).

it is difficult to state the accuracy of the measurements at low frequency. The Young's modulus of PMMA appears in many reference tables, which routinely do not specify the measurement technique. The most common value is between 3 and 3.2 GPa (Boudet & Ciliberto 2000; Fineberg *et al.* 1991; Niemann 1981), however, the reported range is between 2.7 GPa (Williams *et al.* 1968; Barquins & Petit 1992) and 3.7 GPa (Enciclopedia dell'Ingegneri 1971), thus covering both our static and intermediate measurements.

5 CONCLUSION

In conclusion, we find that the static and dynamic moduli define two broad classes comprising different measurement techniques with different accuracy and generally affected by inherent bias with values up to 10 per cent. We can summarize our results by stating that the Young's modulus of Calcare Massiccio measured over nine decades at room conditions is comprised within 10 per cent of an average value of 75 GPa. Because the Poisson's ratio is not expected to change with frequency, we retain the most accurate dynamic value $\nu = (0.28 \pm 0.02)$. No significant dependence on frequency nor on strain amplitude is apparent for Calcare Massiccio in undamaged condition.

Even in the data found in literature (*cf.* Eissa & Kazi 1988) the difference between static and dynamic moduli in brittle rocks is mostly comprised within 10 per cent, which is comparable with the 10 per cent discrepancy between different measurement methods shown here and with the common scatter between different samples. Indeed, in such cases there is no significant frequency dependence in the elastic moduli and we may argue that the simple and widely available seismic methods provide an acceptable estimate of the elastic properties of brittle rocks.

However, the laboratory measurements are generally performed on undamaged small scale samples, while the large wavelength of the seismic investigations (10^2 to 10^4 m) provide estimates of the elastic moduli averaged over larger scales involving a high degree of heterogeneity and damage of the rocks, which could produce a significant anisotropy and frequency dependence. The static response of these rocks to large scale stresses can thus be quite different from that estimated by the seismic measurements and should be further investigated.

However, if we can exclude significant dependence on frequency for the rock in undamaged conditions, the difference between the seismic measurements and the laboratory measurements can be used to estimate the degree of damage (Budiansky & O'Connell 1976; Kachanov 1999).

As an example, the seismic measurements performed in the upper 12 m in the Casavecchia Fratelli quarry¶, where our Calcare Massiccio samples where taken, show a *P*-wave velocity increasing approximately from 1000 to 4500 m s^{-1} with depth, which is significantly lower than our laboratory measurement of 6200 m s^{-1}. This reflects a severe state of cracking and damage of Calcare Massiccio near the surface of the quarry.

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¶Seismic refraction profile performed by Idrogeotec s.n.c., Italy. 12 geophones with frequency range 10–14 Hz were used along a 120 m path.

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