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Static and Dynamic Moduli of the Seismogenic Layer in Italy

By

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Summary

Static and dynamic elastic moduli of *Calcare Massiccio* (mudstone-limestone) have been measured as a function of frequency over nine decades, using three different standard methods: uniaxial 'static' compression, dual cantilever forced oscillations, and measurement of ultrasonic velocities. An accurate critical comparison using the same techniques on a poly-methyl-methacrylate (PMMA) material shows that an unresolvable 10 to 20% bias exists in the two 'low' frequency standard techniques, whereas the ultrasonic measurements are more accurate and reproducible within 5%. No significant frequency dependence is found for *Calcare Massiccio*, which gives a Young's modulus of (75 $\pm$ 7) GPa and a Poisson's ratio of (0.28 $\pm$ 0.02).

Keywords: Calcare Massiccio, static moduli, dynamic moduli, limestone.

1. Introduction

Elastic constants are relevant parameters whenever stresses and strains are considered. The main methods used to measure them are:

a) the *static method*, which is based on the measurement of the deformation induced in a material by the application of a known force;
b) the *forced oscillation method*, which consists of measuring the ratio between force and deformation in an intermediate range of frequency;
c) the *dynamic method*, which implies the measurement of the ultrasonic body wave velocities.

Due to intrinsic anelastic effects, these values do not necessarily coincide. In spite of this, the lack of available data forces one to use them indifferently, albeit with persisting doubts about the reliability of such an operation. In regard to the Earth's crustal rocks, the problem is aggravated by the fact that only dynamic measurements

Table 1. Chemical composition of *Calcare Massiccio* obtained through XRF analysis at the Dipartimento di Scienza della Terra of Bologna, Italy

CaO	50.55%
LOI	43.69%
MgO	5.00%
Fe_2O_3	0.30%
Al_2O_3	0.20%
Sr	170 ppm
Trace elements	0.24%

LOI Loss On Ignition (weight loss after burning at 500 °C for four hours).

are readily provided by seismic waves. As a consequence, their use in modeling is widespread, although its appropriateness is at doubt (Jaeger and Cook, 1969). The aim of the present paper is to ascertain this, by analyzing jointly, over a range of nine frequency decades, a rock and a material, poly-methyl-methacrylate (PMMA), for which a good number of measured data is available at all frequencies, thus guaranteeing against systematic errors in the specific apparatuses used.

The rock analyzed is the *Calcare Massiccio* (a compact homogeneous mudstone-limestone), which is very interesting *per se*, being the typical rock of the seismic focal regions in Italy (Almagro, 2002) where data are not available. The samples were obtained from the *Casavecchia Fratelli* quarry, located in Cagli (Pesaro), central Italy. The chemical composition is reported in Table 1. The apparent density (measured by hydrostatic weighting) is $\rho_A = (2.71 \pm 0.03)\,\mathrm{kg/m^3}$, the density of the grains (obtained by weighting a submicrometric powder in a picnometer) is $\rho_A = (2.84 \pm 0.05)\,\mathrm{kg/m^3}$. This results in a very low porosity: $n = (5 \pm 2)\%$. Isotropy was preliminarly verified by ultrasonic measurements on cubic blocks of 10 cm side. The rocks were tested after resting for 24 hours at room conditions (25 °C, 50% R.H.).

2. Static Moduli

The measurement of the static elastic moduli has been performed following the standard procedure UNI9724[1], by using an AMSLER uniaxial compression machine in the LARM laboratory[2]. Following this procedure, prismatic specimens ($20 \times 5 \times 5$ cm in size) have been submitted to a low compressional displacement rate (about 0.1 mm/min), both in a monotonic load up to failure (three specimens), and by several loading cycles with progressively increasing loads (one specimen). The axial and lateral strains in the middle section of the specimen have been computed by averaging the values measured by orthogonal strain gauges in the middle of the four lateral faces (see the sketch in Fig. 1). The average stress on the same section has been evaluated by dividing the measured total load by the area of the middle section.

Typical stress-strain curves for the monotonic and cyclic tests are shown in Fig. 2. Both cases are consistent with a high degree of linearity of *Calcare Massiccio*. The

[1] Ente Nazionale Italiano di Unificazione – http://www.uni.com – Standard norm: UNI 9724-8:1992 – ICS code: 91.100.15. Stones. Determination of the elastic modulus (uniaxial).

[2] Laboratorio di Resistenza dei Materiali, DISTART, Università di Bologna, Italy.

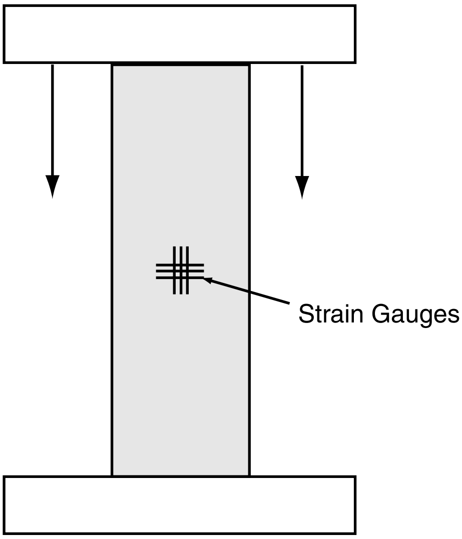

Fig. 1. Loading configuration for the uniaxial compression test

values of the tangent Young's modulus for three monotonic tests, reported in Table 2, exhibit a small dispersion:

$$E = (77 \pm 1.5)\,\text{MPa}$$

This falls within the range of compact limestones, which is comprised between 30 and 90 GPa (Carmichael, 1984). The average tangent Poisson's ratio $\nu = (0.43 \pm 0.01)$ obtained in the monotonic tests appears rather high for a limestone, but this value is apparently not well determined by the static technique since the tangent Poisson's ratio in cyclic test shows a lower value $\nu = 0.30$ (see Table 2).

3. Intermediate Moduli

The measurements in an intermediate frequency range have been performed by a DMA2980 machine in the INFM[3] laboratory. As shown in Fig. 3, a plate of dimensions $60 \times 13 \times 5$ mm is loaded in a Dual Cantilever configuration and undergoes a series of forced oscillations with constant frequency in the range 0.01 to 20 Hz. The machine measures the intensity ratio K (stiffness) and phase shift δ between the force and displacement oscillations, thus providing the complex Young's modulus $\boldsymbol{E} = E' + i\,E''$. The real and the imaginary components, E' and E'', have been evaluated through the equation[4]:

$$E = \frac{k}{F_c}\frac{L^3}{24I}\left(1 + \frac{12}{5}(1+\nu)\left(\frac{d}{L}\right)^2\right) \qquad (1)$$

[3] Istituto Nazionale di Fisica della Materia, Bologna, Italy.
[4] From the Technical Reference Manual of the instrument DMA2980 (http://www.tainstruments.com).

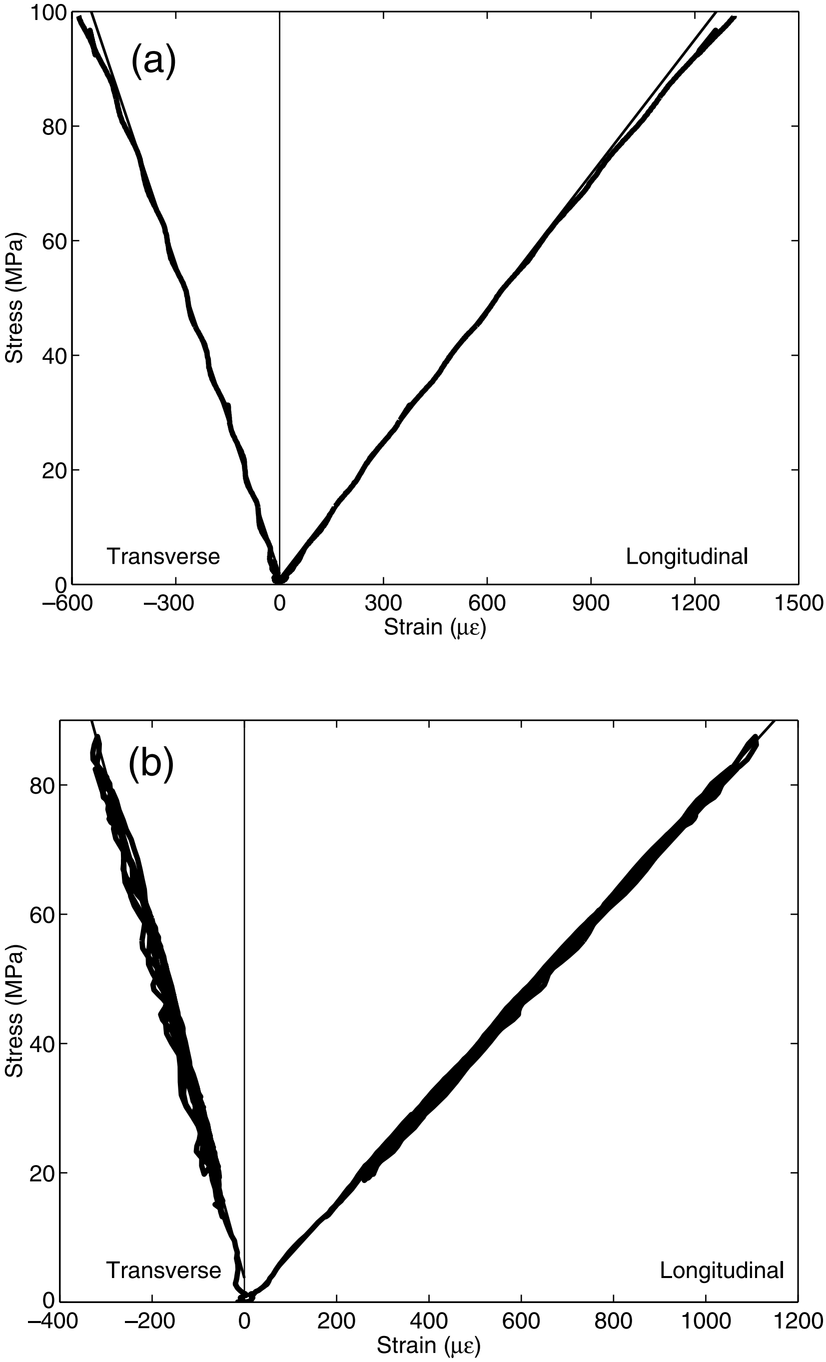

Fig. 2. Stress-strain diagrams for the monotonic test on specimen C_2 (**a**) and the cyclic test on specimen C_1 (**b**). The straight lines represent respectively the tangent modulus and a linear fit in the cycled region

Table 2. Compressive strength and static elastic moduli (longitudinal and transverse Young's modulus along with Poisson's ratio) for the monotonic tests on the three specimens C_2, C_5 and C_6, and the cyclic test on specimen C_1

	σ_c (MPa)	E_l (GPa)	E_t (GPa)	ν (–)
C_2	107.0	78.4	−180	0.43
C_5	120.7	77.0	−179	0.43
C_6	109.1	75.3	−173	0.44
Mean	112	77	−178	0.43
|max−min|	14	3	8	0.01
C_1	87.5	78.8	−260	0.30

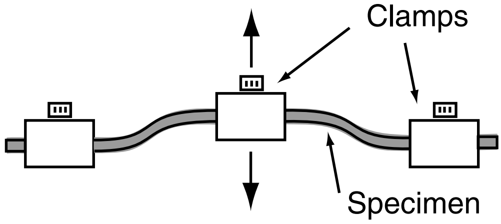

Fig. 3. Dual cantilever forced oscillations

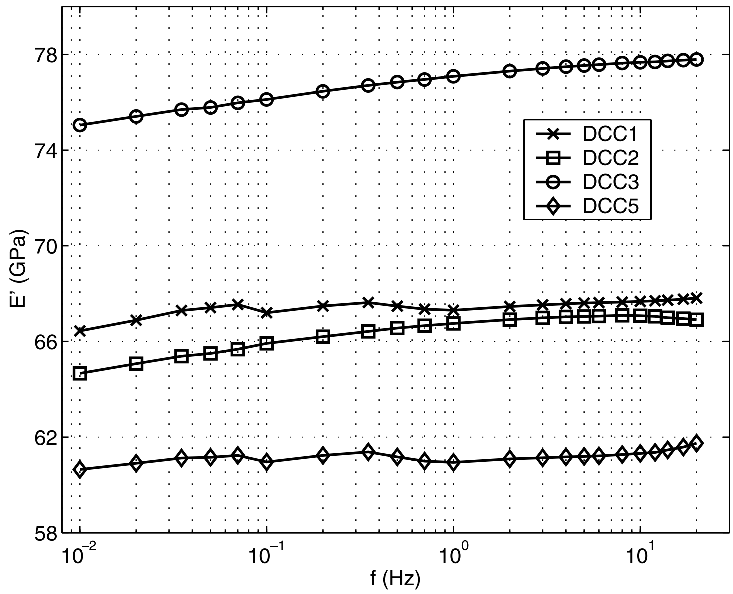

Fig. 4. Real component of the Young modulus as a function of frequency for four *Calcare Massiccio* specimens at the temperature of 25 °C

where k is one of the two components of the measured stiffness ($k' = K\cos\delta$, $k'' = K\sin\delta$), L is the distance between clamps, d and w are the thickness and width of the specimen, $I = wd^3/12$ is the moment of inertia, F_c is a corrective factor determined by finite element analysis, and ν is an estimate of the Poisson's ratio which must be provided independently.

The values of the storage Young's modulus and of $\tan\delta = E''/E'$ calculated on four *Calcare Massiccio* specimens are shown in Figs. 4 and 5. We assumed the value of

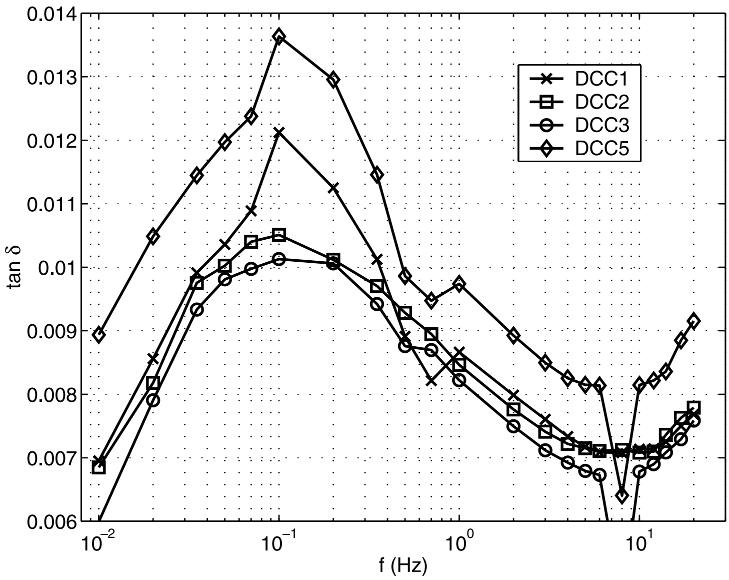

Fig. 5. tan δ as a function of frequency for four *Calcare Massiccio* specimens at the temperature of 25 °C

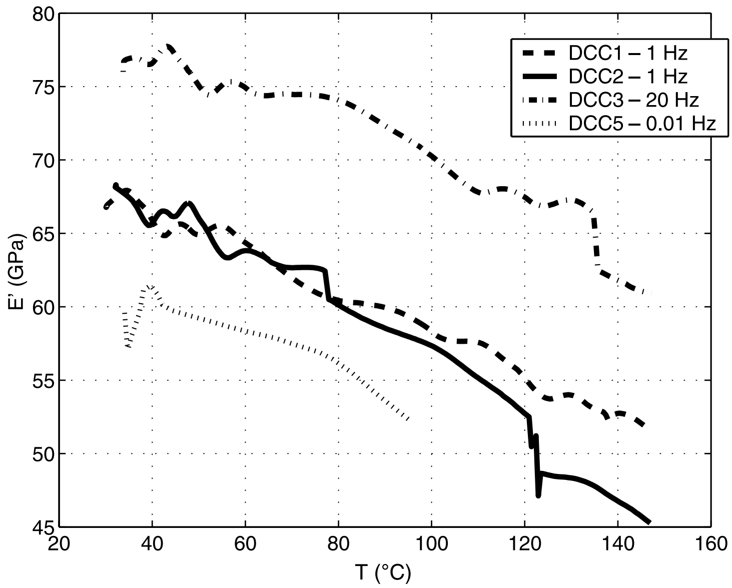

Fig. 6. Effect of temperature on the real component of the Young's modulus of *Calcare Massiccio*

Poisson's ratio $\nu = 0.28$ which results from the dynamic measurements, since the Young's modulus estimated with this method is only weakly affected by this choice. The dependence of Young's modulus on frequency is found to be very weak in the explored range (about 3%), especially when compared with the variability between samples. All the measurements can be well represented by the average value at 1 Hz:

$$E = (68 \pm 6)\,\mathrm{GPa}.$$

The maximum of tan δ, which is evident in all specimens at a frequency of 0.1 Hz, is the signature of some dissipative phenomenon with a relaxation time of about 10 s.

Additional measurements have been performed to investigate the influence of temperature in the range 30–150 °C (see Fig. 6). Ignoring some small oscillations – that cannot be considered significant – as some jumps – which are to be ascribed to internal failures in the specimen –, we can retain an estimate of the average decrease of the Young's modulus with temperature of about $-140\,\mathrm{MPa/°C}$.

4. Dynamic Moduli

The high frequency moduli have been obtained by measuring the velocities v_P and v_S of longitudinal and transverse elastic waves and using the equations (Kuttruff, 1991):

$$E = \rho \frac{v_S^2\left(3v_P^2 - 4v_S^2\right)}{v_P^2 - v_S^2} \tag{2}$$

$$\nu = \frac{v_P^2 - 2v_S^2}{2\left(v_P^2 - v_S^2\right)}. \tag{3}$$

Measurements at 1 Mhz have been made using either two *P*-wave VP-1093 Pinducer transducers in transmission configuration, or a single Panametrics transducer in pulse-echo configuration (V103 for *P* waves, V153 for *S* waves). Measurements at 75 kHz have only been performed for *P*-waves, using custom transducers in the LARM laboratory. The Young's modulus has been evaluated through (Kuttruff, 1991):

$$E = v_P^2 \rho \frac{\left(1+\nu\right)\left(1-2\nu\right)}{\left(1-\nu\right)} \tag{4}$$

using the Poisson's ratio measured at 1 MHz[5].

The results of the measurements on 12 prismatic *Calcare Massiccio* samples ($20 \times 5 \times 5$ cm) are reported in Table 3. It should be noted that, if at 1 MHz the wavelength of 6 mm is small in relation to the sample transverse thickness, this is not the case at 75 kHz for which the wavelength is 4 cm. This could produce waveguide modes resulting in underestimates of the body wave velocity. We verified by measurements on PMMA that this does not happen because the first peak of the direct body wave is well distinguishable before the arrival of the waves reflected by the side walls.

[5] The Poisson's ratio is not expected to depend much on frequency in an isotropic medium, since it is the ratio of two orthogonal deformations that should have a similar dependence on frequency.

Table 3. Measured P and S waves velocities and dynamic moduli evaluated through (2) and (3) when v_S is available, or otherwise through (4) with ν assumed to be the same as the one measured at 1 MHz. The P wave velocity 1 MHz is the average value between pulse-echo and transmission measurements, which differed only by 0.6%

f (kHz)	v_P (m/s)	v_S (m/s)	E (GPa)	ν (−)
1000	6195 ± 150	3427 ± 90	81.4 ± 4.5	0.28 ± 0.02
75	6169 ± 260	–	80.7 ± 7	–

5. Discussion

The values of the Young's modulus given by the three experimental methods are within 20%: a percentage comparable to the typical heterogeneity between different *Calcare Massiccio* samples (see for example Fig. 4).

An apparently surprising result is that the static modulus, $E = (77 \pm 1.5)$ GPa, is higher than the one of the intermediate range, $E = (68 \pm 6)$ GPa, but lower than the dynamic modulus, $E = (81 \pm 5)$ GPa. This apparent inconsistency relates to the fact that the 'static' measurements are not really static, since the tests have a typical duration of about 20 min, and have therefore associated a frequency of $\sim 10^{-3}$ Hz.

Since the latter frequency is just a decade lower than the lowest frequency of the 'intermediate' measurements, the results appear puzzling indeed, considering, moreover, that the intermediate measurements show a very weak positive dependence on frequency (within 3%) in the range from 0.01 to 20 Hz. This combined evidence suggests the presence of a bias in one or both the static and the intermediate method.

To investigate this point further, we used the same techniques and apparatuses on a well known material, PMMA[6], for which a wealth of measurements exist and which, being a rheologic material, exhibits a large variation between static and dynamic moduli.

The static measurements on PMMA gave a value of $E = (3.6 \pm 0.1)$ GPa, again higher than the lower limit of the intermediate measurements, which range from (2.7 ± 0.2) GPa at 0.01 Hz to (3.8 ± 0.2) GPa at 20 Hz with a well defined increasing trend. The dynamic measurements gave a constant value of $E = (6.02 \pm 0.2)$ GPa at both 75 kHz and 1 MHz.

While the dynamic values are very stable and highly consistent with independent measurements, which range between 6.0 and 6.3 GPa (Boudet and Ciliberto, 2000; Filippi et al., 1999; Ferry, 1980), it is difficult to state the accuracy of measurements at lower frequency. The Young's modulus of PMMA appears in many reference tables, which usually do not specify the measurement technique adopted. The most common value is between 3 and 3.2 GPa (Boudet and Ciliberto, 2000; Fineberg et al., 1991; Niemann, 1981), but the reported range is between 2.7 GPa (Williams et al., 1968; Barquins and Petit, 1992) and 3.7 GPa (Enciclopedia dell'Ingegneria, 1971), thus covering both our static and intermediate measurements. As an exception, Koppelmann (1958) gave a complete curve of the Young's modulus of PMMA for frequencies ranging from 10^{-4} to 10^3 Hz, suggesting a static limit $E = 3$ GPa, values in the

[6] We used cast PMMA (Plexiglas) produced by Röhm and Haas Italia Srl.

intermediate range between 3.2 GPa at 0.01 Hz to 3.9 GPa at 20 Hz, and an upper limit of 5 GPa at 1 kHz, in which the trend is still positive.

In conclusion, our static and intermediate measurements are inconsistent, but comprised in the typical range found in literature. Since these measurements have been performed using standard techniques, and well calibrated apparatuses, we argue that the differences are tied to the details of the measurement techniques themselves. An important issue is the different strain amplitude involved in the three tests: $\varepsilon \sim 10^{-3}$ for static measurements; $\varepsilon \sim 10^{-5}$ for dual cantilever; $\varepsilon \sim 10^{-7}$ for ultrasonic waves. However, no systematic strain amplitude effect is noticed.

Summing up, it is typical to find an unresolvable 10 to 20% bias between different standard measurement techniques. While the large frequency dependence of the elastic moduli of PMMA is however well apparent, this is not the case for non rheologic materials like *Calcare Massiccio*. In this case there is no evidence of frequency dependence in the elastic moduli, and the use of the dynamic method appears to provide the easiest and most accurate figure.

For *Calcare Massiccio* we can conclude, therefore, that the elastic moduli sampled over nine decades are within 20% of an average value of 75 GPa, with no evident systematic effect, and that Poisson's ratio is equal to (0.28 ± 0.02).

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