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Stick-slip in the peeling of an adhesive tape: evolution of theoretical model

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Abstract

This paper analyses the dynamic equations representing the peeling dynamics of an adhesive tape from a rotating support. Three degrees of freedom are considered. Speed jumps are shown to be possible and are then introduced into the dynamics by discontinuous operators. Differences with previous models are studied, with regard also to the eventuality of chaotic orbits. The observed metastability of the stationary branches is accounted in an early catastrophe model. An analogy between the sudden jumps of the crack speed and the abrupt phase transitions of a van der Waals' fluid is developed with the aim of suggesting a possible statistical interpretation of fracture dynamics.

Keywords: Adhesive tape; Pressure sensitive; Destructive testing by peel; Fracture dynamics; Stick-slip

1. Introduction

The everyday experience shows that in a given range of temperatures and velocities, the peeling of an adhesive tape is jerky with emission of a characteristic noise. In previous works [1–4] the peeling was studied when a constant traction velocity V_0 is imposed onto the free end by the action of an electric motor (Fig. 1). The limit between the adhesive tape ribbon and the free end may be seen as a crack tip that propagates with speed v.

Three different modes of peeling were observed, relative to the velocity applied: at slow speed V_0, the tape was peeled regularly with the crack propagation speed $v = V_0$ and both the speed v and the peeling force F increased with the imposed speed V_0. It was shown that the strain energy release rate G varied as a power function of the crack propagation speed V_0. The same phenomenon was found at high speeds with a considerable rise of the peeling force with increasing velocity. Between these two modes, a phenomenon of self-sustained oscillations (stick-slip) already described by Aubrey and Sherriff [5] was observed, i.e. the peeling of the tape becomes jerky with the emission of a characteristic noise.

Some dynamic models have been proposed in order to explain the nature of the stick-slip dynamics and to understand the role of the main parameters like the value of the pull speed V_0, the length L of the free ribbon between the reel and the motor (this affecting the elasticity constant k), the moment of inertia I of the rotary support, the values of temperature T and humidity ratio HR.

A 'two variables' model [1–3] (G, v) produced good results when applied to the initial part of the stick-slip region (at slow speed), but when the traction speed is increased the oscillations become very irregular and do not correspond with theoretical predictions. A first three variables model (F, v, α) introducing the variations of the peeling angle θ and the position α of the crack tip (Fig. 1) was proposed by Hong and Yue [4] and the numerical simulations indicated the presence of chaotic orbits (positive Lyapunov exponents were found).

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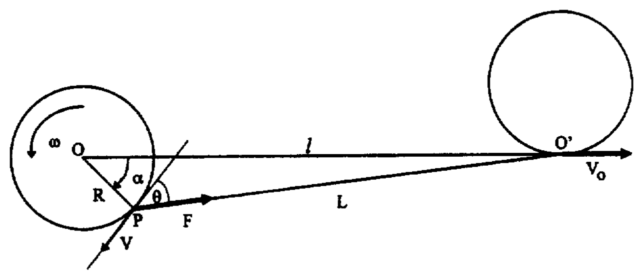

Fig. 1. *System geometric description.* The ribbon is driven by the engine (on the right) with traction speed V_0. The fracture line at point P is subjected to a force F at an inclination θ to the fracture plane. The apparent movement of the fracture point (indicated by $\dot{\alpha} \cdot R$) is the result of a noncompensation between the rotational movement $\omega \cdot R$ and the crack speed v. L is the length of the free portion and l the distance OO′.

In our previous work [6] we reported a different experiment where the tape was unrolled by the action of a load hooked to the free extremity (constant pull force). The interesting results concerning the observation of speed jumps and the hysteretical behaviour of the fracture's advance have stimulated further studies and led to some modifications of the physical interpretation which seem to get us closer to the real nature of the stick-slip dynamics in peeling.

The main aim of this paper is to expose three different steps which represent successive possible improvements of the physical interpretation and modelling.

2. Three-variables model and dynamic equations

Let us first review a three-variables continuous model, following the path of Hong and Yue [4]. System geometric configuration and variables are shown in Fig. 1, where I is the moment of inertia and ω is the rotational velocity of the support, F the applied force, v the crack speed, V_0 the traction speed (which is maintained constant by the action of an electric motor), α the position of the fracture line (point P in Fig. 1), θ the peeling angle, L the length of the free ribbon and l the distance OO' in Fig. 1. Let's call k the linear elasticity constant of the free ribbon (affected by the length L).

For this system, using the approximated relations (when length $l \gg R$) $l \approx$ L, $\alpha + \theta \approx \pi/2$, $\sin\theta \approx \cos\alpha$, $\cos\theta \approx \sin\alpha$, we obtain the following set of dynamic equations (*System* 1):

$$\begin{cases} F \cdot (1 - \sin\alpha) = F_0(v), \\ I \cdot \dot{\omega} = F \cdot R \cdot \sin\alpha, \\ \dot{F} = k \cdot [R \cdot \sin\alpha \cdot \dot{\alpha} - (v - V_0)], \\ R \cdot \dot{\alpha} = v - \omega . R. \end{cases} \tag{1}$$

This *System* 1 is the same as that of Hong and Yue [4] except for the elimination of θ and for some different computational assumptions about signs and approximations.

The relation:

$$F \cdot (1 - \sin\alpha) = F_0(v) \tag{2}$$

derives from the experimental relation $F.(1 - \cos\vartheta) = F_0(\mathrm{v})$, obtained by Rivlin [7] which gives the crack speed when a constant force F is applied to the ribbon at an inclination θ to the adhesion plane. The relation $F_0(v) = G/b$ (where b is the lateral width of the tape and G the strain energy release rate) is determined empirically by measuring $F(v)$ for $\theta = \pi/2$ (normal peeling for an adhesive tape roller). Although Eq. (2) was obtained in conditions of stationary speed, in literature it is normally used also as a dynamic relation between F, v and α.

In this way it can be used to eliminate α, thus obtaining a set of three differential equations in the three variables (F, v, ω):

$$\begin{cases} \dot{F} = k \cdot \left[\dfrac{F - F_0(v)}{F} \cdot (v - \omega \cdot R) - (v - V_0) \right], \\ \dot{v} = \dfrac{1}{\frac{\mathrm{d}F_0}{\mathrm{d}v}(v)} \cdot \left[\dot{F} \cdot \dfrac{F_0(v)}{F} - F \cdot \sqrt{1 - \left(1 - \dfrac{F_0(v)}{F}\right)^2} \right. \\ \qquad \left. \cdot \dfrac{v - \omega \cdot R}{R} \right], \\ \dot{\omega} = \dfrac{R}{I} . (F - F_0(v)), \end{cases} \tag{3}$$

which is valid for $\mathrm{d}F_0/\mathrm{d}v(v) \neq 0$ and F $\neq 0$.

Considering a traction velocity V_0 which does not correspond with a maximum or minimum of $F_0(v)$, we can verify that Eq. (3) allows only one fixed point $F = F_0(V_0)$, $v = V_0$, $\omega = V_0/R$ for $\alpha = 0$ (i.e. $\theta = \pi/2$). This point is stable if $\mathrm{d}F_0/\mathrm{d}v(V_0) > 0$. In the case of a negative slope, the fixed point is unstable and it is surrounded by a stable limit cycle, which is assumed to represent the stick-slip motion (Fig. 2b).

More exactly when the pull speed V_0 crosses the critical value v_C, the fixed point becomes unstable, but surrounded by a stable limit cycle; the occurring bifurcation is the hypercritical Hopf bifurcation (Fig. 2a) whose order parameter is ε.

$$\varepsilon = -\frac{1}{2} \cdot \frac{R}{\sqrt{k \cdot I}} \cdot \left(\frac{\mathrm{d}F_0}{\mathrm{d}v}\right)_{v = V_0}.$$

The limit cycles are initially period one, but when V_0 grows through the intermediate range $v_C < V_0 < v_A$ (Fig. 2b) the limit cycle becomes multiperiodic and also presents chaotic orbits when V_0 is comprised into a narrow range far from v_C (positive maximum Lyapunov exponents were found [4]).

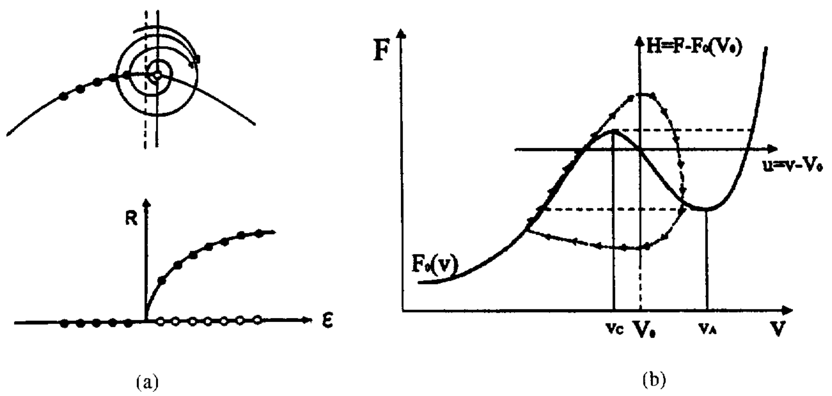

Fig. 2. (a) *Hypercritical Hopf bifurcation.* When the order parameter ε becomes positive, a stable fixed point splits into an unstable fixed point surrounded by a stable limit cycle of radius $R \propto \sqrt{\varepsilon}$. (b) *Limit cycle on the curve* $F = F_0(v)$. These cycles appear when the traction speed V_0 crosses the critical value v_C. The first part of the cycle is superposed to the slow branch of the curve, then it separates due to inertial effects.

This was taken as an interpretation of the great irregularities observed in the stick-slip cycles. We may notice again that stick-slip cycles are considered to be continuous and that the jerky behaviour is considered to be the limit situation when the inertia of the rotary support can be ignored [2].

3. Catastrophic propagation model

In this paper we propose a different interpretation of the stick-slip dynamics in peeling. By working with imposed load [6, 8], the experimental evidence is that when the force crosses the critical value $F_C = F_0(v_C)$ (Fig. 3) a speed jump occurs together with an acoustic emission.

The crack speed rises immediately up to a figure one thousand times greater, so that the crack proceeds much quicker than the pull point, causing a decrease of the free ribbon extension and consequently of the effective force acting on the fracture line. When the force falls under the second critical value F_A (corresponding to v_A, in Fig. 3) a second speed jump occurs towards slow crack propagation and the ribbon starts stretching again.

This seems to give an interesting interpretation of the stick-slip dynamics present when the traction speed V_0 is imposed. The intermediate speed range $v_C < v < v_A$ would practically be banned for the crack speed v, so that when the traction speed V_0 is in this range, the crack speed v undergoes a series of discontinuous dynamic cycles BCDAB, as shown in Fig. 3.

This interpretation was first hinted at in earlier works [1, 3] (for a two variables model) and then discarded by Maugis [2] who thought the speed jumps were in contradiction with the presence of the inertial effects due to the rotary support of the tape (see the second equation of the System 1).

The two-variables model used by Maugis [2] takes the approximation of ignoring the variations of the crack

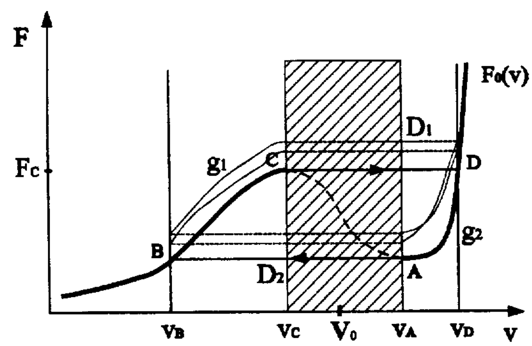

Fig. 3. *Discontinuous dynamic cycles* (*stick-slip*). The fracture undergoes a continuous evolution g_1 along the segment BC, then a sudden speed jump D_1 between C and D (dashed line), then a second continuous evolution g_2 along DA and a second jump D_2 between A and B. The cycles do not lie exactly on the curve $F_0(v)$ because of the variations of the fracture line's position α (see Eq. (2)). This also allows the cycles to be not to be necessarily closed (a period-two cycle is shown in the picture).

position denoted by α. This way the last equation of System 1 becomes $v = \omega \cdot R$, thus introducing a nonrealistic strict bond between the crack speed and the rotational speed of the reel. This forced Maugis [2] to discard the speed jumps of the crack speed because they would imply discontinuities in the rotational speed of the reel.

Going back to the three variables model, we can see in the last equation of System 1 that a discontinuity of the crack speed v can occur together with a continuous evolution of the rotational speed ω provided that we also have a discontinuity in $\dot{\alpha}$ i.e., if the apparent position of the fracture line starts a sudden precession. This can be observed while unrolling a tape (try to produce a stick-slip cycle with a common cellotape!).

We should notice that the speed jumps will occur also if the support is a rigid table with infinite inertia and are not just a limit situation of zero-inertia (try it by pulling the ribbon while holding tight the reel). We think that there are no other reasons to reject the speed jumps, which are so near to the evidence and which were a basic part of the stick-slip interpretation before Maugis' model [2, 3]. The speed jumps represent a catastrophic behaviour and must be introduced into the dynamic equations by two discontinuous operators D_1 and D_2 which instantly increase (or decrease) the crack speed when the critical value v_C (or v_A) is crossed, leaving the values of F and ω unaffected. The evolution on the two continuous branches simply follows the classical differential set (System 1).

Note that the evolution of the phase point $x = (F, v, \omega)$ does not lie on the curve $F_0(v)$ in Fig. 3 because of the variations of α (Eq. (2)). The diagram of F versus v is just a projection of the three dimensional phase space.

If we denote with g_1^t and g_2^t the evolution operators on the two continuous branches (solutions of System 1) and we start a cycle from point x_B we can express the evolution as follows (see Fig. 3):

$$x_C = g_1^t(x_B) \rightarrow T_1 = T_1(x_B),$$
$$x_D = D_1(x_C) \rightarrow x_C = (F_C, v_C, \omega_C) \rightarrow x_D = (F_C, v_D, \omega_C), \quad (4)$$
$$x_A = g_2^t(x_D) \rightarrow T_2 = T_2(x_D),$$
$$x_B = D_2(x_B) \rightarrow x_A = (F_A, v_A, \omega_A) \rightarrow x_B = (F_A, v_B, \omega_A),$$
$$x_B' = D_2(g_2^{T_2}(D_1(g_1^{T_1}(x_B)))) = M(x_B).$$

In this way the evolution operator M for one complete cycle in the three dimensional phase space can be constructed by a composition of two branches of continuous evolution and two speed jumps (Fig. 3):

$$M = D_2 \circ g_2^{T_2} \circ D_1 \circ g_1^{T_1}, \quad (5)$$
$$x_B^{N+1} = M(x_B^N).$$

Thus we can obtain a recursive law which can be used to study the nature of the orbits, with the aim to verify the presence of chaotic orbits in fracture dynamics. We must observe that this interpretation does not allow for any limit cycle or Hopf bifurcation, and that the eventual presence of chaos would not depend on the properties of the intermediate speed range which is uncoupled from the dynamics. An experimental hint at this theory lies in the fact that when the traction speed exceeds the critical value v_C, the stick-slip regime starts abruptly with a large amplitude.

4. Early catastrophe interpretation

We have also looked further into the physical nature of the speed jumps and attempted to explain certain experimental behaviours which do not match with any of the previous models.

Our previous works [6, 8] show that the terminal part of the slow branch is indeed metastable when working with imposed load, i.e., a little perturbation induces an early crack starting the stick-slip regime also if the load has not reached the critical value F_C. It was also observed [5] that the two stable branches correspond with different fracture modalities (plastic-brittle behaviour). This guides us to simplify the peeling dynamics just to what is observed: the dissipation curve (Fig. 3) is indeed made of the two single branches with positive slope corresponding to two different fracture modalities. There is no way of obtaining a fracture propagation with a speed included in the intermediate range, i.e., there is no realistic negative slope branch, but just a forbidden region. When the pull force crosses the critical value F_C the slow fracture modality becomes unstable and the system abruptly starts the fast modality which is stable around these values of the force.

When the traction speed V_0 is comprised in the forbidden range, the crack speed is forced to undergo the stick-slip cycles, mediated by the elastic property of the free ribbon. The ribbon undergoes a series of subsequent extensions and relaxations because the crack speed v is always found to be smaller or bigger than the traction speed V_0 which cannot be reached.

An interesting behavioural analogy is given by the van der Waals' model (Fig. 4): an S-shaped curve was expected for a monophasic gas at constant temperature, but experience shows that only the two negative slope branches correspond with natural systems and are related to different states of association: liquid and gas. Regions CF and EA were found as metastable, while no monophasic system exists when the volume is in the intermediate region AC. If we take a gas to point F and increase the pressure with enough care, we can still observe a gas, but a minimal perturbation induces a sudden phase transition to liquid. If we reach the point C, the phase transition is certain. The transition is not indeed instantaneous: when the metastable monophasic gas loses stability there is a transitory out of equilibrium situation before the system is reorganised in the liquid phase.

This leads us to develop a statistical interpretation of fracture dynamics: if we consider the macroscopic advance of the fracture line as the combination of a huge number of microfractures in its neighbourhood (which is a realistic hypothesis), Eq. (2) is interpreted as a state equation for the fracture and represents the possible values of the macroscopic crack speed when the fracture experiences a force F at a temperature T. The S-shaped curve is indeed an isothermal curve and its negative slope branch simply does not exist because no macroscopic crack speed is observed inside this range.

The two positive-slope branches represent two different modes (phases) of fracturing [5], the last part of each branch being metastable [6]. When the top of a branch is

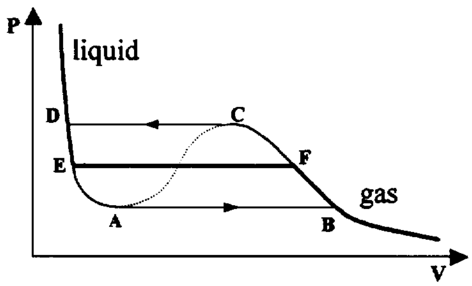

Fig. 4. *Isothermal curves for the Van der Waals' model.* The thick line shows a realistic liquid–gas phase transition. The flat line EF corresponds to coexistence of the two phases. The thin line EACF corresponds to Van der Waals' predictions for a monophasic system. Branches EA and CF are shown to exist as metastable states, while the dashed branch AC can not be observed. If a gas reaches point C (or even before) it undergoes a sudden phase transition to liquid. The same (reversed) holds for a liquid reaching point A.

reached (or even before if a perturbation occurs) the first fracture modality loses stability towards the other one, so that we have a speed jump.

If speed jumps can effectively be considered, as some sort of abrupt phase transitions in a statistic 'microfractures system', we deduce that they are not really instant but they take a transitory time (in the evidence very short) during which the system is out of equilibrium and is not represented by Fig. 2.

For the mathematical model, when a perturbation induces an early speed jump, we must anticipate the action of the discontinuous jump operator in the dynamic solution, which becomes this way more complex. In addition, if Eq. (2) is indeed a state equation, nothing assures that it can be used as a dynamic constraint: its effectiveness would be reduced to slow variations of the pull force, which can be considered as 'quasistatic' in relation to the characteristic transitory time (relaxation time) of the statistic system.

In fact the stick-slip cycles are very fast (from 100 to 700 cycles/s) and therefore it is difficult to think of a quasistatic evolution. For high rates, early transitions could become the ordinary behaviour for each cycle, the point of transition remaining random.

This *early catastrophe's model* is just a hypothesis born from the observation of the metastabilities, but aptly explains some experimental anomalies which were not understood before.

For example it was observed [1] that when the stick-slip rate increases, a fall in the amplitude of the force oscillations occurs, which is consistent with the easier occurrence of early catastrophes at high rates. Moreover, for high stick-slip rates the typical matt tracks left on the ribbon vanish [1]. These tracks are left when the cycles pass by a little cohesive region near point B in Fig. 3. If we have an early jump during the descent of DA, the little cohesive region would not be covered, thus motivating the absence of the matt track. A further hint at the statistical interpretation is also given by the strong dependence on temperature of the state equation (through $F_0(v)$, see Refs. [1, 3]).

5. Summary and conclusion

The two-variables model [1–3] is good to investigate the stability of the fixed points corresponding to stationary crack advance, but it is insufficient to explain the nature of the stick-slip phenomenon because it does not admit the presence of speed jumps. Using a three-variables model we have shown that speed jumps (which are very easy to observe on a common tape) are indeed possible because they are compensated by sharp variations of the peeling angle.

This has led us to recover the old interpretation [1] of the stick-slip cycles as discontinuous cycles made of a jerky alternation between slow and high speed fracture advancement. A new mathematical model was then developed to describe such cycles in a three dimensional phase space, alternating phases of continuous evolution under certain differential equations [4] and sharp speed jumps that occur when the critical values of the crack speed or peeling force are reached.

This shows that the intermediate speed region is never visited by the crack speed and can be considered as not existing because never observed and not necessary to explain the phenomenon. The characteristic curve of the fracture is simply just made of two positive slope branches which correspond to two different fracture modalities (plastic–brittle behaviour). If this is correct, the hypothesis of a Hopf bifurcation and the interpretation of stick-slip cycles as continuous limit cycles should be discarded and also the research on chaotic orbits [4] should be revised by evaluating the Lyapunov exponents on the new kind of discontinuous orbits.

Speed jumps are well interpreted mathematically by catastrophic events, but what is their physical meaning? What happens to the fracture during a jump? We believe that the fracture advance should be interpreted as the result of a huge number of microfractures which produce a macroscopic coherent front line which can advance only with speed smaller or higher than the intermediate forbidden region. When the speed reaches the critical values, the macroscopic coherence is lost and the system is quickly reorganised to move at a very different speed, whose value corresponds to the same force on the other branch.

We are then probably facing a statistical rather than purely dynamical system and care for the consequences in the mathematical treatment. Eq. (2) is better interpreted as a state equation and should be used dynamically only if the variations are slow relative to the statistic relaxational times, which seems not to be always the case.

The study of the propagation under constant load [6] has shown a metastability of the stationary propagation near the critical points. The fracture presents some sort of an early catastrophe which is very similar to the abrupt phase transition of a real gas when carefully compressed over saturation. When the stick-slip rate grows, an early catastrophe seem to be the ordinary end of each cycle, thus strongly contributing to cause the irregularity observed in the force registrations, and motivating the fall in amplitude observed.

The aim of the present paper is just to show that the classical dynamical way of treating fractures seems to be insufficient to deal with fast stick-slip cycles and that a statistical interpretation would be very useful to understand the real nature of fracture propagation.

Work is in progress to see if it is possible to construct a statistical model able to preview the shape of the state equation (Eq. (2)), the duration of the transitory time and the reliability of the state equation as a dynamic constraint. Some new experiments are also being projected to provide a long series of experimental results which will undergo some correlation tests in order to inquire the real dimensionality of the orbits in phase space and to state if it is possible to treat the system in a simply dynamic way.

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